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Topology optimization of geometrically nonlinear structures based on an additive hyperelasticity technique

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Abstract

This paper presents a simple but effective additive hyperelasticity technique to circumvent numerical difficulties in solving the material density-based topology optimization of elastic structures undergoing large displacements. By adding a special hyperelastic material to the design domain, excessive distortion and numerical instability occurred in the low-density or intermediate-density elements are thus effectively alleviated during the optimization process. The properties of the additional hyperelastic material are established based on a new interpolation scheme, which allows the nonlinear mechanical behaviour of the remodelled structure to achieve an acceptable approximation to the original structure. In conjunction with the adjoint variable scheme for sensitivity analysis, the topology optimization problem is solved by a gradient-based mathematical programming algorithm. Numerical examples are given to demonstrate the effectiveness of the proposed method.

Keywords: Geometrical nonlinearity; Topology optimization; Hyperelastic material; Sensitivity analysis

1. Introduction

Topology optimization of linear elastic and nonlinear (geometrical and/or material) structures has been extensively investigated in the literature [1–4]. It is widely recognized that, while topology optimization based on linear elastic structural responses is valid for a variety of problems, it often has a poor performance when subjected to large deformation but small strain. In these situations, geometrical nonlinear analysis for structural behaviours should be included in the optimization process.

In recent years, topology optimization considering geometrical nonlinearity has been rapidly developed and has been applied to wide fields including mechanics, compliant mechanisms and multidisciplinary problems. Yuge et al. [5] firstly used the homogenization method to investigate the layout design of structures subjected to nonlinear

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deformations. Buhl et al. [6], Gea and Luo [7] compared the optimal topologies obtained by linear modelling and geometrically nonlinear modelling. Their numerical examples showed that differences in these two topologies are generally small under small displacement, but can be large in certain cases involving buckling or snap-through effects. Based on the SIMP (Solid Isotropic Microstructure with Penalty) approach and the nonlinear finite element analysis, Pedersen et al. [8] proposed a synthesis method of large-displacement compliant mechanisms and Sigmund [9] further developed the topology optimization of thermo-electrical micro actuators. Bruns and Tortorelli [10] investigated the topology optimization problem of compliant mechanisms considering geometric and material nonlinearities. By determining the critical load level directly, Kemmler et al. [11] introduced the instability phenomena into the topology optimization process. Bruns et al. [12], Bruns and Sigmund [13] studied the topology optimization problems of mechanisms that exhibit snap-through behaviours. In addition, some powerful numerical techniques, such as the (probabilistic or non-probabilistic) reliability analysis method and the meshfree method, have also been incorporated into the optimal layout design of geometrically nonlinear structures by many authors [14–17].

As the literature survey reveals, the use of material interpolation schemes such as the power-law penalization in the SIMP is currently the most popular approach in solving topology optimization problems of geometrically nonlinear structures. In this framework, material properties are modelled as a function of the relative material density; crisp black-and-white solutions can be obtained by penalizing intermediate densities. However, the consideration of the geometric nonlinearity may cause the so-called "element instability" phenomenon due to excessive mesh distortion of the weak-material elements. When the finite element analysis is used, the tangent stiffness matrix of distorted mesh will become indefinite or even negative definite, which leads to non-convergence of the Newton–Raphson equilibrium iterations.

Since the element instability phenomenon occurs only in the nonlinear analysis part but not in the optimization part, there are two commonly used techniques, namely the "convergence criterion relaxation" [6,8,9] and the "element removal" [18], for alleviating mesh distortion in the Newton–Raphson process. The former method excludes nodes surrounded by minimum-density (or void) elements from the convergence criterion. Non-convergence difficulty caused by minimum-density elements is thus avoided. In the latter method, an element will be removed in the finite element analysis if it takes minimum values of the relative density, and can reappear in the subsequent optimization process by using density filtering techniques. Due to the fact that ignoring void nodes or elements has little influence to the structural response, these two methods are reliable and usually work effectively if the instability occurs in material phases with minimum or close to minimum stiffness. Mesh distortion, however, may also occur in intermediate-density material during the optimization process, especially when the structure undergoes relatively large deformations. In such a case, the convergence criterion relaxation method or the element removal method is not a complete remedy to overcome the element instability.

In addition, Yoon and Kim [19] proposed an element connectivity parameterization method, in which solid finite elements are controlled by many zero-length links and the structural layout is represented by inter-element connectivity. This method has also been extended to three-dimensional problems [20] and stress constrained problems [21]. Kawamoto [22] proposed a Levenberg–Marquardt method to replace the Newton–Raphson method in the geometrically nonlinear analyses and thus stabilizes the optimization process. Recently, Wang et al. [23] proposed a new energy interpolation scheme to alleviate the numerical instability in the low stiffness region.

In this work we focus on the numerical instability of the topology optimization of geometrically nonlinear elastic structures. A simple and effective "additive hyperelasticity technique" is proposed to circumvent element instability phenomenon by adding an extremely soft hyperelastic material to weak material elements. Based on this idea, a new interpolation scheme for hyperelastic material is built, the adjoint variable scheme for design sensitivities is obtained, and finally the topology optimization is solved using the Method of Moving Asymptotes (MMA) [24]. Numerical examples are presented to demonstrate the numerical accuracy and effectiveness of the proposed method.

2. Topology optimization of geometrically nonlinear structures

We consider a general topology optimization problem of geometrically nonlinear structures, which aims to seek the minimum compliance design in a given design domain. In the numerical implementation, the design domain is meshed into finite elements and the relative densities of elements are taken as the design variables. By using the SIMP approach [25] to build a penalized model for artificial isotropic materials, the original discrete 0–1 programming