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Complex variational mode decomposition for signal processing applications

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ABSTRACT

Complex-valued signals occur in many areas of science and engineering and are thus of fundamental interest. The complex variational mode decomposition (CVMD) is proposed as a natural and a generic extension of the original VMD algorithm for the analysis of complex-valued data in this work. Moreover, the equivalent filter bank structure of the CVMD in the presence of white noise, and the effects of initialization of center frequency on the filter bank property are both investigated via numerical experiments. Benefiting from the advantages of CVMD algorithm, its bi-directional Hilbert time-frequency spectrum is developed as well, in which the positive and negative frequency components are formulated on the positive and negative frequency planes separately. Several applications in the real-world complex-valued signals support the analysis.

1. Introduction

In modern disciplines, complex-valued signals are encountered in a wide variety of applications, such as structural health monitoring, sensor array processing as well as biomedical sciences and physics. For the case of bivariate (or complex valued) data, several extensions to traditional real-valued signal decomposition method have been developed so far. For example, bivariate EMD (BEMD) has been proposed in [1], and it has been used to wind turbine condition monitoring in [2], because of its advantages for information fusion. Motivated by the bivariate framework in the BEMD method, a complex local mean decomposition algorithm was proposed in [3]. However, this class of complex extension methods can only be used on the assumption that complex signals are proper or circular. A proper complex random variable is uncorrelated with its complex conjugate, and a circular complex random variable has a probability distribution that is invariant under rotation in the complex plane [4]. However, there are many cases where proper and circular random signals are very poor models of the underlying physics. Therefore, another class of complex-extension of the EMD (CEMD) was introduced in [5], which cleverly used the relationship between the positive and negative frequency components. Currently, the CEMD has been applied to perform the fusion of data from multiple and heterogeneous sources [6]. However, CEMD can not guarantee the same number of intrinsic mode functions (IMFs) across real and imaginary data channels, which is a major requirement in real-world applications.

Variational mode decomposition (VMD) was recently proposed in [7] as an alternative to the empirical mode decomposition (EMD) for the separation of composite real-valued time series into respective modes. It has been pointed out that VMD is theoretically much better founded compared to the sequential iterative sifting of EMD, because VMD is based on a clear variational model and the resulting minimization steps perform concurrent mode extraction in an intuitive way. It was also demonstrated in [7] VMD over EMD has some advantages on tones separation and is less sensitive to noise and sampling. Based on these characteristics,

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VMD was applied to detect rub-impact fault of the rotor system [8]. Nevertheless, the VMD has been presently developed only for real-valued data, its wider ranges of applications in signal processing and related fields are limited. Motivated by the work [5], a new extension of the VMD designed to handle complex-valued time series is developed in this work. To further shed light on its performance, the behavior of CVMD in the presence of white Gaussian noise is also analyzed, which will be of great benefit to applications of the CVMD.

Numerous signals in science and engineering exhibit time-varying oscillatory behavior that is not possible to characterize adequately by conventional Fourier analysis. Time-frequency analysis can identify the signal frequency components, reveals their time variant features, and is an effective tool to extract machinery health information contained in nonstationary signals. Various time–frequency (TF) analysis methods have been proposed and applied to machinery fault diagnosis [9]. Moreover, it should be noted that interest in time-frequency analysis of multichannel data has also recently been growing with multivariate data driven algorithms that directly exploit multichannel interdependencies [10] and [11]. A multivariate TF algorithm based on synchrosqueezing transform that generated a compact TF representation of multichannel signals was developed in [12]. Motivated by the Hilbert-Huang spectrum [13], the full Hilbert time-frequency spectrum of CVMD is thus developed in this work as well, in which the positive and negative frequency components are formulated on the positive and negative frequency planes separately.

The rest of this work is organized as follows: Section 2 briefly recalls the VMD algorithm developed in [7]. The proposed complex extension of the VMD and the corresponding filter bank property as well as its Hilbert spectrum are all proposed in Section 3. Three applications based on the CVMD method and its Hilbert spectrum are addressed in Section 4. Conclusion is presented in Section 5.

2. Variational mode decomposition

The VMD can non-recursively decompose a real-valued multi-component signal f into a discrete number of quasi-orthogonal band-limited sub-signals u_k with the specific sparsity properties of its bandwidth in the spectral domain [7]. Each mode is compact around a center pulsation ω_k and its bandwidth is estimated using $\mathcal{H}^1$ Gaussian smoothness of the shifted signal. For the convenience of following discussions, let us commence by calling these modes obtained by VMD as band-limited IMFs (BLIMFs) in this work. BLIMFs are different from IMFs defined in EMD technique according to the numbers of extrema and zerocrossings as well as zero-mean binding conditions. VMD first turns the real valued mode u_k into an analytic signal u_k^+ with single sided frequency spectrum

$$u_k^+(t)=\left(\delta(t)+\frac{j}{\pi t}\right)*u_k(t) \tag{1}$$

The single sided spectrum is then shifted down to 0-frequency baseband, and eventually the effective bandwidth of such a signal is obtained using the $\mathfrak{L}_2$-norm of the time derivative. Actually, the generalization in the proposed CVMD is to decompose the complex signal into two single-sided frequency spectra using an ideal band-pass filter, which can be found in detail in the following section. The VMD method is written as a constrained variational problem [7]:

$$\min_{\{u_k\},\{\omega_k\}}\left\{\sum_{k=1}^{K}\|\partial_t[u_k^+(t)e^{-j\omega_k t}]\|_2^2\right\}, \text{subject to} \sum_{k=1}^{K}u_k(t)=f(t) \tag{2}$$

The constraint in Eq. (2) can be addressed by introducing a quadratic penalty and Lagrangian multipliers $\lambda(t)$. The augmented Lagrangian is given as follows

$$\mathcal{L}(\{u_k\},\{\omega_k\},\lambda)=\alpha\sum_{k=1}^{K}\|\partial_t[u_k^+(t)e^{-j\omega_k t}]\|_2^2 + \left\|f(t)-\sum_{k=1}^{K}u_k(t)\right\|_2^2+\left\langle\lambda(t), f(t)-\sum_{k=1}^{K}u_k(t)\right\rangle \tag{3}$$

where α denotes the balancing parameter of the "data-fidelity" constraint. The corresponding unconstrained problem in Eq. (3) is then solved using the Alternate Direction Method of Multipliers (ADMM) [14]. All the modes with $i<k$ gained from solutions in the Fourier domain are updated in nature by Wiener filtering using a filter tuned to the current center frequency on the positive part of the spectrum (i.e.,$\omega\geq 0$), that is

$$\hat{u}_k^{n+1}(\omega)=\frac{\hat{f}(\omega)-\sum_{i<k}\hat{u}_i^{n+1}(\omega)-\sum_{i>k}\hat{u}_i^{n}(\omega)+\frac{\hat{\lambda}(\omega)}{2}}{1+2\alpha(\omega-\omega_k^n)^2} \tag{4}$$

where the center frequency ω_k^n is accordingly updated as the center of gravity of the corresponding mode's power spectrum $\hat{u}_i^{n+1}(\omega)(\omega\geq 0)$. Due to the embedded Wiener filtering in the algorithm, VMD is much more robust for sampling and noise. The ω_k^{n+1} is computed as follows

$$\omega_k^{n+1}=\frac{\int_0^\infty \omega|\hat{u}_k^{n+1}(\omega)|^2 d\omega}{\int_0^\infty |\hat{u}_k^{n+1}(\omega)|^2 d\omega} \tag{5}$$