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A method for detection and characterisation of structural non-linearities using the Hilbert transform and neural networks

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ABSTRACT

This paper presents a method for detection and characterisation of structural non-linearities from a single frequency response function using the Hilbert transform in the frequency domain and artificial neural networks. A frequency response function is described based on its Hilbert transform using several common and newly introduced scalar parameters, termed non-linearity indexes, to create training data of the artificial neural network. This network is subsequently used to detect the existence of non-linearity and classify its type. The theoretical background of the method is given and its usage is demonstrated on different numerical test cases created by single degree of freedom non-linear systems and a lumped parameter multi degree of freedom system with a geometric non-linearity. The method is also applied to several experimentally measured frequency response functions obtained from a cantilever beam with a clearance non-linearity and an under-platform damper experimental rig with a complex friction contact interface. It is shown that the method is a fast and noise-robust means of detecting and characterising non-linear behaviour from a single frequency response function.

1. Introduction

In recent years, as a result of the wide applicability of well-developed modal testing, the research focus in structural dynamics has turned to the identification of non-linear systems. Non-linear system identification is usually divided into three stages – detection, characterisation and parameter quantification [1]. Detection, which aims to find non-linear behaviour in a system response, is the first step towards establishing a structural model with a good predictive accuracy. The purpose of characterisation is to specify the type, spatial location and mathematical form of the non-linearities which have been detected in the structure. It is very important to detect and characterise the non-linear elastic and dissipative mechanisms involved prior to parameter quantification. Without a precise understanding of non-linear behaviour, the identification process is bound to fail [2].

A number of different approaches for detection and characterisation have been developed in recent decades [2–4]. Many of these approaches are based on the violation of basic linear principles, for instance the widely used homogeneity method [5], or

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Nomenclature

Symbol	Description
b	clearance size parameter [m]
c	linear damping coefficient [N m^{-1} s^{-1}]
c_{nl}	non-linear (quadratic) damping coefficient [N m^{-2} s^{-1}]
f	frequency [Hz]
f_0	undamped natural frequency [Hz]
$f(x)$	activation (transfer) function
$F(t)$	excitation force [N]
F	amplitude of excitation force [N]
$\mathbf{F}(t)$	excitation force vector [N]
$g_{\mathrm{nl}}(x, \dot{x})$	non-linear restoring force [N]
$G(\omega)$	difference between an FRF and its Hilbert transform
$\mathbf{G}_{\mathrm{nl}}(\mathbf{x}, \dot{\mathbf{x}})$	non-linear restoring force matrix [N]
$H(\omega)$	frequency response function – receptance [m N^{-1}]
$\mathrm{HTD}^{(n)}$	nth Hilbert transform describer
k	linear stiffness coefficient [N m^{-1}]
k_{cl}	clearance stiffness coefficient [N m^{-1}]
k_{nl}	cubic stiffness coefficient [N m^{-3}]
$\mathbf{K}$	stiffness matrix [N m^{-1}]
m	mass [kg]
$m(\omega)$	weight function
$M_H^{(n)}$	nth-order spectral moment
n_{h}, n_{sl}	number of harmonic components and spectral lines
$\mathbf{M}$	mass matrix [kg]
NLI_i	ith non-linearity index
$R_{H\tilde{H}}(\Delta\omega)$	cross-correlation
$\mathbf{S}$	scatter matrix
$\mathbf{U}$	matrix of principal component directions
w_{ij}^{k}	weight between ith and jth node of kth layer
x, $\dot{x}$, $\ddot{x}$	displacement [m], velocity [m s^{-1}], acceleration [m s^{-2}]
$x_i^{(k)}$	signal at node i in layer k
$\mathbf{x}_s$	vector of samples
$\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$	displacement [m], velocity [m s^{-1}], acceleration [m s^{-2}] vectors
$\mathbf{X}_0$	displacement due to static forces [m]
$\mathbf{X}_j^c$, $\mathbf{X}_j^s$	cosine and sine harmonic coefficient [m]
$\mathbf{z}$	principal component scores
α, β	Rayleigh's proportional damping coefficients
γ_1	centre of gravity
γ_2, γ_3	moment of inertia about ω-axis and with respect to origin
Δf	frequency resolution [Hz]
$\Delta\omega$	angular frequency lag [rad s^{-1}]
μN_{f}	Coulomb friction force [N]
σ	standard deviation
$\boldsymbol{\Sigma}$	diagonal matrix of relative contributions
$\varphi(\omega)$	phase [rad]
ω	angular frequency [rad s^{-1}]
$\omega_{\min}$	minimum angular frequency [rad s^{-1}]
$\omega_{\max}$	maximum angular frequency [rad s^{-1}]
Ω	substitution of ω in integrand
$\bullet^*$	normalised attributes
$\tilde{\bullet}$	Hilbert transformed attributes
$\bar{\bullet}$	arithmetic mean
$\bullet^T$, $\bullet^{-1}$	matrix transpose and inverse
$\mathcal{H}\{\bullet\}$	Hilbert transform operator
$\Re\{\bullet\}$	real part
$\Im\{\bullet\}$	imaginary part
pv	Cauchy principal value
sgn $(\bullet)$	sign function
ANN	Artificial Neural Network
FORSE	FOrced Response SuitE
FRF	Frequency Response Function
MDOF	Multi Degree Of Freedom
PCA	Principal Component Analysis
PCS	Principal Component Scores
SDOF	Single Degree Of Freedom

on the distortion of frequency response functions (FRFs) [1] or describing function inversion [6]. Such methods usually require the time-consuming measurement process of many FRFs while relying on an experienced operator to detect and characterise non-linear behaviour correctly. In addition, measured noise in FRFs frequently causes significant problems, leading to the inapplicability of some techniques [7].

The Hilbert transform is a well-known technique used in many fields of engineering, especially in signal processing [8]. Although most of its applications are performed in the time domain, the Hilbert transform can also be applied in the frequency domain. Not only does the Hilbert transform provide a fast and effective means of detecting non-linear behaviour on the basis of a measured FRF [9,5], but it also provides limited insight into the type of non-linearities [2,1].

However, the Hilbert transform still has several limitations. Perhaps the main limitation, which is also associated with all detection methods that look for distortion in measured signals, is the lack of an established methodology to determine if the deviations observed in an FRF are statistically significant. Therefore, the diagnosis of non-linearity requires expert judgement [2]. This limitation can be overcome by using artificial neural networks (ANNs) as suggested in [10,11]. The approach proposed in [10] allows localisation and characterisation of the type of non-linearity, but it requires a full spatial model of the structure with all possible combinations of non-linearities. Unfortunately, this makes the approach difficult to use in practical applications. In contrast, the method proposed in [11] does not require the spacial model of the structure for characterisation of the type of non-linearity. It uses the gain of FRFs as training data for ANNs and therefore a large number of training cases are required, while the complex nature of FRFs is not fully taken into account due to missing phase information.

A similar method as in [11] has been used in this study. In order to avoid the need for a vast set of training data, non-

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