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Softening behavior of quasi-brittle material under full thermo-mechanical coupling condition: Theoretical formulation and finite element implementation

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Abstract

In this article we introduce a new thermo-damage model, which is capable of modeling the softening behavior of quasi-brittle under loading condition including temperature change. The theoretical formulation and numerical solution procedure to solve the problem in the most general case of full thermo-mechanical coupling are presented.

Keywords: Thermo-mechanical coupling; Thermo-damage; Softening; Adiabatic split procedure

1. Introduction

The thermo-mechanical inelastic behavior of material was considered by many research works. We can mention several representative works of Simo and Miehe [1], Armero and Simo [2], Ibrahimbegovic et al. [3] on thermo-plasticity behavior for small deformation and of Ibrahimbegovic et al. [4] on thermo-plasticity behavior for large deformation. The continuum thermo-damage behavior of quasi-brittle material was also studied by Baker and de Borst (see [5]), but with no numerical implementation aspects.

The softening behavior of material under mechanical loading was also considered by many researchers, for example the article of Armero [6], Oliver [7], Ortiz [8] or of Ibrahimbegovic et al. [9,10] for plastic material and Brancherie et al. [11–13] for damage material. There is not as many previous research works concerning the thermo-plastic softening behavior of material. We can mention the work of Armero and Park [14] on the plastic softening behavior of a shear layer and the paper of Runesson et al. [15] on the plastic softening of material under adiabatic condition which leads to different points of view related to temperature versus heat flux discontinuity at the localization condition. We mention our recent work on thermo-plastic softening behavior of materials (see [16]) trying to settle the issue of the type of discontinuity for temperature field and address numerical implementation.

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There are not many papers considering the thermo-damage softening of material, though it is necessary for determining the behavior of many practical quasi-brittle materials such as concrete or masonry, neither the theoretical analysis nor the numerical solution.

The main goal of this work is to provide both theoretical formulation and the finite element implementation for localized failure of quasi-brittle materials with damage model under fully coupled thermo-mechanical condition. The outline of the paper is as follows. In the next section, we introduce a new thermo-damage model, which is capable of modeling the thermo-mechanical softening behavior of the material. The discrete approximation of the problem and its numerical solution using the finite element method for the problem are presented in Sections 3 and 4. Several illustrative examples are presented in Section 5, followed by a conclusion in Section 6.

2. General framework

2.1. General continuum thermodynamic model

Several authors contributed to the thermo-damage coupling model, we can cite among others Baker and de Borst [5], or Ngo et al. [17].

The starting point is the local form of the first principle of thermodynamics for the case of thermo-mechanical inelastic response [18]:

$$r - \nabla \mathbf{q} = -\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} + \dot{e}(\boldsymbol{\varepsilon}, \eta^e) \tag{1}$$

where r is the internal heat supply, $\mathbf{q}$ is the heat flux, $\boldsymbol{\sigma}$ is the stress field, $\boldsymbol{\varepsilon}$ is the strain field, e is the internal stored energy and η^e is the reversible part of entropy ($\dot{\bullet}$ denotes the time rate of the variable $\bullet$).

By following [2,1,5], the entropy is considered as the composition of the reversible part (or "elastic" entropy) and irreversible part (or "inelastic" entropy):

$$\eta = \eta^e + \eta^d. \tag{2}$$

By the Legendre transformation, the internal stored energy can be expressed in terms of the free energy ψ:

$$e = \psi + \eta^e \vartheta \tag{3}$$

where ϑ denotes the absolute temperature of the media.

In thermo-damage framework, we can assume as the most generally accepted [5,17] that $\psi(\boldsymbol{\varepsilon}, \vartheta, \mathbf{D}, \xi)$ is the function of the state variables: the total strain $\boldsymbol{\varepsilon}$, the temperature ϑ, the compliance tensor $\mathbf{D}$ and the hardening variable ξ.

The Clausius–Duhem inequality for the model is written as:

$$0 \le D_{\text{int}} = \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} + \vartheta \dot{\eta} - \dot{e} = \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} + \vartheta \dot{\eta} - \dot{\psi} - \dot{\eta}^e \vartheta - \eta^e \dot{\vartheta} \tag{4}$$

$$0 \le D_{\text{int}} = \left(\boldsymbol{\sigma} - \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}}\right) \cdot \dot{\boldsymbol{\varepsilon}} - \left(\eta^e + \frac{\partial \psi}{\partial \vartheta}\right) \dot{\vartheta} + \left(\vartheta \dot{\eta}^e - \vartheta \dot{\eta}^e\right) + \vartheta \dot{\eta}^d - \frac{\partial \psi}{\partial \mathbf{D}} \dot{\mathbf{D}} - \frac{\partial \psi}{\partial \xi} \dot{\xi}. \tag{5}$$

In the case of "elastic" process, where $\dot{\mathbf{D}} = 0$ and $\dot{\xi} = 0$, the Clausius–Duhem inequality becomes equal and therefore, the constitutive equations for the stress and the "elastic" entropy can be established:

$$\boldsymbol{\sigma} = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}} \tag{6}$$

$$\eta^e = -\frac{\partial \psi}{\partial \vartheta} \tag{7}$$

and the dissipation equation can also be written:

$$D_{\text{int}} = -\frac{\partial \psi}{\partial \mathbf{D}} \dot{\mathbf{D}} - \frac{\partial \psi}{\partial \xi} \dot{\xi} + \vartheta \dot{\eta}^d. \tag{8}$$

Also, by applying Eq. (3) and the constitutive equations (6), (7), the first principle of thermodynamics can be rewritten:

$$r - \nabla \mathbf{q} = -\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} + \dot{\psi} + \dot{\eta}^e \vartheta + \eta^e \dot{\vartheta}$$