Mechanical Systems and Signal Processing 82 (2017) 32–55

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Global identification of stochastic dynamical systems under different pseudo-static operating conditions: The functionally pooled ARMAX case

J.S. Sakellariou [a,*], S.D. Fassois [a,b]

[a] Stochastic Mechanical Systems & Automation (SMSA) Laboratory, Department of Mechanical Engineering & Aeronautics, University of Patras, GR 265 04 Patras, Greece
[b] Department of Mechanical Engineering, Khalifa University of Science, Technology & Research, (KUSTAR), PO Box 127788, Abu Dhabi, UAE

ARTICLE INFO

Article history:
Received 1 January 2016
Received in revised form 27 March 2016
Accepted 6 May 2016
Available online 2 June 2016

Keywords:
Global model identification
Stochastic identification
Functionally pooled models
ARMAX models
Asymptotic analysis
Railway suspension modelling

ABSTRACT

The identification of a single global model for a stochastic dynamical system operating under various conditions is considered. Each operating condition is assumed to have a pseudo-static effect on the dynamics and be characterized by a single measurable scheduling variable. Identification is accomplished within a recently introduced Functionally Pooled (FP) framework, which offers a number of advantages over Linear Parameter Varying (LPV) identification techniques. The focus of the work is on the extension of the framework to include the important FP-ARMAX model case. Compared to their simpler FP–ARX counterparts, FP–ARMAX models are much more general and offer improved flexibility in describing various types of stochastic noise, but at the same time lead to a more complicated, non-quadratic, estimation problem. Prediction Error (PE), Maximum Likelihood (ML), and multi-stage estimation methods are postulated, and the PE estimator optimality, in terms of consistency and asymptotic efficiency, is analytically established. The postulated estimators are numerically assessed via Monte Carlo experiments, while the effectiveness of the approach and its superiority over its FP–ARX counterpart are demonstrated via an application case study pertaining to simulated railway vehicle suspension dynamics under various mass loading conditions.

1. Introduction

This study concerns the broad problem of global identification of stochastic dynamical systems under different pseudo-static operating conditions. The problem is important, with applications including structural systems (bridges, sea vessels and trains) vibrating under different loading conditions [1,2], structures vibrating under different environmental or boundary conditions [3,4], rotating machinery dynamics under different speeds [5], aircraft dynamics at various altitudes or

Abbreviations: AR, X, MA, AutoRegressive, eXogenous, Moving Average; ARX, AutoRegressive with eXogenous excitation; ARMAX, AutoRegressive Moving Average with eXogenous excitation; FP–ARMAX, Functionally Pooled ARMAX; OLS, Ordinary Least Squares; PE, Prediction Error; 2SLS, 2 Stage Least Squares; LMS, Linear Multi Stage; ML, Maximum Likelihood; BIC, Bayesian Information Criterion; AIC, Akaike Information Criterion; SPP, Samples per Parameter

* Corresponding author.
E-mail addresses: sakj@mech.upatras.gr (J.S. Sakellariou), fassois@mech.upatras.gr (S.D. Fassois).
URL: http://www.smsa.upatras.gr (J.S. Sakellariou).

http://dx.doi.org/10.1016/j.ymssp.2016.05.002
0888-3270/

Nomenclature[1]

k	scheduling variable
$x_k[t]$	excitation signal for the k operating condition
$y_k[t]$	response signal for the k operating condition
$w_k[t]$	model innovations for the k operating condition
$e_k[t]$	one-step-ahead prediction error (residuals) for the k operating condition
$\mathcal{N}$	normal distribution
na, nb, nc	AutoRegressive (AR), eXogenous (X) and Moving Average (MA) orders
pa, pb, pc	dimensionality of AR, X and MA functional subspaces (equal to p if $pa = pb = pc$)
$E\{\cdot\}$	statistical expectation
$\gamma_w(k, l)$	innovations cross correlation between operating conditions k and l
$\sigma_w^2(k)$	innovations variance as a function of the scheduling variable
$a_{i,j}, b_{i,j}, c_{i,j}$	AR, X, MA coefficients of projection
$G_j(k)$	j basis function
$\boldsymbol{\theta}$	coefficients of projection vector
$\bar{\boldsymbol{\theta}}$	augmented parameter vector including innovations variance
M	number of excitation-response signal pairs used for FP-ARMAX identification
N	signal length in samples for each individual operating condition
$\mathcal{B}$	backshift operator ($\mathcal{B}^j \cdot u[t] = u[t-j]$)
$\otimes$	Kronecker product
$_o$	subscript designating actual (true) system
$\operatorname{plim}_{N\to\infty}$	probability limit operator
$\xrightarrow{d}$	convergence in distribution
$\xrightarrow{p}$	convergence in probability
$Cov\{\cdot,\cdot\}$	covariance between two random quantities
$o(x)$	function that tends to zero faster than x

flight conditions [6,7], and many more.

In a recent paper by the authors [8], a proper *Functionally Pooled framework (FP)* suitable for overcoming the difficulties of alternative (such as LPV-type [5,9]) approaches and obtaining global stochastic models characterized by optimal statistical accuracy, was introduced. The postulated framework includes the concept of Functional Pooling, Functionally Pooled models, and optimal estimation techniques. Although the form of FP models may appear to resemble that of LPV models [10], they have a number of important differences: all signal pairs are treated as a single entity, the number of estimated parameters is minimal, potential cross-correlations among the signal pairs are accounted for, and the estimation is accomplished in a single step and has been analytically and numerically confirmed to achieve optimal accuracy [8].

Yet, thus far, FP models have been mainly used in their AutoRegressive with eXogenous excitation (ARX) form, employing simple estimation techniques, and mainly from an application point of view in studies pertaining to the modelling of composite structures under various temperatures [3] and the modelling of aircraft dynamics under different flight conditions and configurations [6,7]. A main application has also been on damage localization and magnitude estimation, as part of the broader Structural Health Monitoring (SHM) problem [11–16].

The *goal* of the present article is the extension of the FP framework presented in [8] in order to: (i) Include the more complex, but much more general and practically important Functionally Pooled AutoRegressive Moving Average with eXogenous excitation (FP-ARMAX) family of models, (ii) postulate proper Prediction Error (PE), Maximum Likelihood (ML), and multi-stage estimation methods, (iii) analytically investigate the PE estimator *consistency* and asymptotic *efficiency* properties which pertain to the achievement of optimal accuracy, (iv) numerically assess the postulated estimators via Monte Carlo experiments, and (v) present an application case study where FP–ARMAX modelling is used for estimating the dynamics, in the form of global modal characteristics (natural frequencies and damping ratios), of a simulated railway vehicle suspension under various mass loading conditions (corresponding to different numbers of passengers). Comparisons demonstrating the estimation accuracy improvement over that of local models used in the context of LPV–type identification are also made, along with comparisons demonstrating the improvement achieved over FP-ARX modelling. The study constitutes a broad extension of an early conference paper on the topic by the authors [17].

Some remarks on the importance, advantages, and difficulties associated with the postulated FP-ARMAX models – which may be traced back to their conventional counterparts [18, p. 93] – are now in order:

(a) Like in the conventional case, the incorporation of ARMAX type models within the FP framework is very important as these add significant flexibility in capturing the structure and characteristics of a wide range of external noise signals (see the comments pertaining to conventional ARMAX models in [19–24]).

(b) The FP-ARMAX type models employed are characterized by Chebyshev type II [25, pp. 773–782] parameter variation, and the postulated estimation methods are based on a corresponding polynomial operator algebra. Yet, it

[1] *Important conventions*: Bold-face upper/lower case symbols designate matrix/column-vector quantities, respectively. Matrix transposition is indicated by the superscript ^{T}. A functional argument in brackets designates function of an integer variable; for instance $x[t]$ is a function of normalized discrete time ($t = 1, 2, \ldots$). The conversion from discrete normalized time to analog time is based on $(t-1)T_s$, with T_s standing for the sampling period. A hat designates estimator/estimate of the indicated quantity; for instance $\hat{\boldsymbol{\theta}}$ is an estimator/estimate of $\boldsymbol{\theta}$. Tilde designates sample quantity; for instance $\tilde{\sigma}^2$ designates sample variance. For simplicity of notation, no distinction is made between a random variable and its value(s).