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Nonlinear performance of continuum mechanics based beam elements focusing on large twisting behaviors

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Abstract

In this paper, we present the nonlinear formulation and performance of continuum mechanics based beam elements, in which fully coupled 3D behaviors of stretching, bending, shearing, twisting, and warping are automatically considered. The beam elements are directly degenerated from assemblages of 3D solid elements under the assumptions of Timoshenko beam theory. Therefore, cross-sectional discretization is possible and the elements can model complicated 3D beam geometries including curved and twisted geometries, varying cross-sections, eccentricities, and arbitrary cross-sectional shapes. In particular, the proposed nonlinear formulation can accurately predict large twisting behaviors coupled with stretching, bending, shearing, and warping. Through various numerical examples, we demonstrate the geometric (and material) nonlinear performance of the continuum mechanics based beam elements.

Keywords: Nonlinear analysis; Finite element method; Beam element; Large twisting; Warping; Lateral buckling

1. Introduction

Beams are widely used structural members in various engineering fields including marine, mechanical, civil engineering, and aerospace engineering fields [1–5]. Recently, beam applications have rapidly extended from classical metallic structures to nano- and bio-structures [6–8], in which the finite element method is a tool that has been dominantly adopted for analysis and design. In order to encompass these new applications, the modeling capability and nonlinear performance have become increasingly important in the finite element analysis of beams.

For a long time, considerable efforts have been made regarding beam theories and the development of beam finite elements; see Refs. [2,3,9–20] for a review of the existing research. Based on classical beam theories, recent works have focused on developing high performance beam finite elements for linear and nonlinear analyses. As a result, the modeling and analysis capabilities have been continuously improved; see Refs. [21–29] for examples. However, compared with other kinematics, relatively small improvements have been made in the large twisting kinematics of beams.

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It is well known that isoparametric beam elements are degenerated from 3D solid elements [1]. Because the beam elements are based on 3D continuum mechanics, they can easily represent general 3D curved geometries including fully coupled complete strains, and the formulation is simple and straightforward. Despite these significant advantages, the original isoparametric beam elements can generally only consider rectangular cross-sections. While several studies have been undertaken in attempts to overcome this limitation in the isoparametric beam elements [4,30–33,27,34], the resulting beams do not clearly model arbitrary beam cross-sections.

Recently, Yoon et al. [35] proposed the concept of continuum mechanics based beam elements, which are degenerated from assemblages of 3D solid elements. Therefore, these beam elements can be considered to be a direct extension of isoparametric beam elements. However, unlike the isoparametric beam elements, the continuum mechanics based beam elements can easily model complicated 3D beam geometries including curved and twisted geometries, varying cross-sections, eccentricity, and arbitrary cross-sectional shapes by incorporating cross-sectional meshes. Furthermore, the beam elements can represent fully coupled behaviors among bending, shearing, stretching, twisting, and warping.

The objective of this paper is to develop a general nonlinear formulation of continuum mechanics based beam elements and present their nonlinear performance focusing on large twisting behaviors. The total Lagrangian formulation is employed and, unlike other beam elements [22–25], the Wagner effect is automatically (implicitly) considered in the beam formulation due to the continuum mechanics based formulation, which results in the capability for large twisting analyses.

The novel aspects of the proposed beam element formulation are summarized as follows:

- The formulation is simple and straightforward.
- Seven degrees of freedom (DOFs) are used at each beam node in order to ensure the inter-elemental continuity of the warping displacements.
- The formulation can manage complicated 3D beam geometries including curved and twisted geometries, varying cross-sections, eccentricity, and arbitrary cross-sectional shapes [35,36].
- The complete tangent stiffness matrix is obtained because the finite rotation of the director vectors is precisely included up to a quadratic order.
- The large twisting and lateral buckling behaviors can be accurately calculated. In the Green–Lagrange strain, the Wagner strain is included implicitly.

In the following sections, the nonlinear kinematics of the continuum mechanics based beams is described. Then, the strain measures are introduced from the given kinematic description and the assumed strain technique used for locking reduction. Next, the incremental equilibrium equation is presented based on the total Lagrangian formulation. Finally, the novelties of the continuum mechanics based beam finite element are demonstrated using well established numerical examples that focus on large twisting behaviors.

2. Large displacement kinematics

The continuum mechanics based beam elements are degenerated from assemblages of 3D solid finite elements [35]. In this section, the large displacement kinematics of the continuum mechanics based beams is presented. In the following formulations, a superscript (or subscript) t is employed to denote time; however, in the static nonlinear analyses considered in this study, t is a dummy variable that indicates the load levels and incremental variables rather than the actual time as in dynamic analyses [1].

Consider a q-node continuum mechanics based beam element that consists of n sub-beams in the configuration at time t, as depicted in Fig. 1. Allowing warping displacements, the geometry interpolation of the sub-beam m (shaded in Fig. 1) is given by

$$ {}^t\mathbf{x}^{(m)} = \sum_{k=1}^{q} h_k(r)\,{}^t\mathbf{x}_k + \sum_{k=1}^{q} h_k(r)\,\bar{y}_k^{(m)}\,{}^t\mathbf{V}_{\bar{y}}^k + \sum_{k=1}^{q} h_k(r)\,\bar{z}_k^{(m)}\,{}^t\mathbf{V}_{\bar{z}}^k + \sum_{k=1}^{q} h_k(r)\,f_k^{(m)}\,{}^t\alpha_k\,{}^t\mathbf{V}_{\bar{x}}^k, \tag{1} $$

in which ${}^t\mathbf{x}^{(m)}$ is the material position vector at time t, $h_k(r)$ is the 1D shape function at beam node k, ${}^t\mathbf{x}_k$ is the position of beam node k at time t, ${}^t\mathbf{V}_{\bar{x}}^k$, ${}^t\mathbf{V}_{\bar{y}}^k$, and ${}^t\mathbf{V}_{\bar{z}}^k$ are the unit director vectors at time t and are normal to each other, $\bar{y}_k^{(m)}$ and $\bar{z}_k^{(m)}$ denote the position in the beam cross-section at beam node k, $f_k^{(m)}$ is the warping function at beam node k, and ${}^t\alpha_k$ is the corresponding warping degree of freedom at beam node k at time t; see Refs. [35,36] for