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Exact free vibration analysis for mechanical system composed of Timoshenko beams with intermediate eccentric rigid body on elastic supports: An experimental and analytical investigation

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The purpose of this article is to investigate the changes in the magnitude of natural frequencies and their associated modal shapes of Timoshenko beam with respect to different system design parameters. This beam includes an intermediate extended eccentric rigid mass mounted on two elastic segments. The equilibrium equations which govern the transverse and rotational motions are derived. The application of the developed system frequency equation is demonstrated by several illustrative examples. Several end and intermediate conditions are considered. The influence of, rotary inertia, shear deformation, axial load, eccentric mass and elastic segments step ratio on the system natural frequencies and mode shapes are conducted. Several sets of new results are presented. Comparison of the present model results with the experimental data for shaft integrated with intermediate rigid mass demonstrates the accuracy of the analysis in practical applications. The present model is valid for several industrial applications, such as mechanical, structural, naval and for wider range of applications.

1. Introduction

Timoshenko [1] was the first who presented the vibration problem of beams with rotary inertia and shear deformation effect. Huang [2] investigated the frequency equations and normal modes of free flexural vibrations of uniform beams including the effect of shear deformation and rotary inertia for classical end conditions. Sato [3] investigated the governing equations for vibration and stability of a Timoshenko beam subjected to an axial load using Hamilton's principle. Farghaly [4] derived an exact frequency equation for uniform cantilever Bernoulli–Euler beam with an elastically mounted non concentrated tip mass. Farghaly and Shebl [5] studied the vibration and stability of axially loaded Timoshenko beam carrying end masses of finite length and elastically supported against rotation and translations.

Kopmaz and Telli [6] considered the transverse vibration of a system consisting of a rigid body carried by two uniform beams of different flexural rigidity, length and pinned at the beam ends. The center of mass of this rigid body is located at its middle. Naguleswaran [7], Banerjee and Sobey [8] and Ilanko [9] presented three comments on the discontinuity conditions between the beam and rigid mass discussed by Kopmaz and Telli [6]. In addition Kopmaz and Telli illustrated through their

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Nomenclature

A	cross-section area of the beam.
a, b	polynomial roots
c_1,c_2	distance from k_6 and k_5 to points of attachment
c_1^*, c_2^*	c_1/L, c_2/L respectively
e	distance between the mass center of gravity and the point of attachment
e^*	ratio defined as e/L.
E	Young's modulus of elasticity
f	frequency (Hz)
G	shear modulus of rigidity
I	moment of inertia of the beam cross section about the neutral axis.
J_r	rotational moment of inertia of the intermediate mass.
J_r^*	ratio $\left(J_r/\rho_1 A_1 L^3\right)$
k	shear deformation shape coefficient
k, ϕ	elastic stiffness.
L	length of the beam (between points 1 & 4).
L_r^*	ratio $\left(L_r/L\right)$.
m_t	total mass of the beam.
m_r	intermediate mass.
m_r^*	Non-dimensional intermediate mass $m_r/\rho_1 A_1 L$.
P	axial load.
p^{*2}	axial load parameter ($PL^2/E_1 I_1$).
r_1^{*2}	rotary inertia parameter $\left(I_1/A_1 L^2\right)$.
s_1^{*2}	shear deformation parameter $(E_1 r_1^{*2}/G_1 \acute{k}_1)$.
Y	non-dimensional lateral deflection.
x, y, ψ	system co-ordinate of the beam.
Z_1^*, Z_4^*	non-dimensional stiffness parameters defined as $k_1 L^3/E_1 I_1$ and $k_4 L^3/E_1 I_1$ respectively.
Z_5^*, Z_6^*	non-dimensional stiffness parameters defined as $k_5 L^3/E_1 I_1$ and $k_6 L^3/E_1 I_1$ respectively.
γ_r^*	set of non-dimensional terms defined as in Eq. (9d).
γ_1^*, γ_2^*	set of non-dimensional terms defined as in Eq. (9a, f).
δ_1, δ_2	parameter defined as in Eqs. (A6 a, b)
ϵ_r^*	set of non-dimensional terms defined as in equation (9 c).
ϕ_1, ϕ_4	end rotational spring stiffness.
ϕ_5, ϕ_6	rotational spring stiffness.
Φ_1^*, Φ_4^*	non-dimensional rotational spring parameters defined as $\phi_1 L/E_1 I_1$ and $\phi_4 L/E_1 I_1$ respectively.
Φ_5^*, Φ_6^*	non-dimensional rotational spring parameters defined as $\phi_5 L/E_1 I_1$ and $\phi_6 L/E_1 I_1$ respectively.
λ_4^*	frequency parameter $\left(\rho_1 A_1 L^4 \omega^2/E_1 I_1\right)$.
ω	circular frequency (rad/s).
ψ	slope due to bending.
$L\Psi$	non-dimensional slope due to bending.
μ	parameter defined as L_1/L.
ν	Poisson's ratio.
ρ	mass density of the beam material (kg/m^3).
θ_r^*	set of non- dimensional terms defined as in equation (9 e).
θ_1^*, θ_2^*	1st set of non- dimensional terms defined as in equation (9 b, g).
$'$	1st derivative w.r.t. x or ζ.
$''$	2nd derivative w.r.t. x or ζ.
$'''$	3rd derivative w.r.t. x or ζ.
$''''$	4th derivative w.r.t. x or ζ.
C	Clamped (fixed) support.
F	Free support.
P	Pinned (hinged) support.
S	Slide (guided) support.
BET	Bernoulli–Euler theory.
TBT	Timoshenko beam theory.
vs	10 E−12.
vl	10 E+12.

author's reply [10] some interesting points which were carried out and very useful corrections were reported. Su and Banerjee [11] investigated the free vibration of frame works consisting of two-part beam-mass system. An exact dynamic stiffness matrix for the frame elements was developed from the free vibration theory of Bernoulli–Euler beam. Ilanko [12] presented how the transcendental dynamic stability functions can be used to determine the natural frequencies of a simply supported beam system carrying a rigid body satisfying the partial differential equation governing the flexural motion of Bernoulli–Euler beams exactly. Naguleswaran [13] investigated the transverse vibration of stepped Bernoulli–Euler beam carrying a non-symmetrical rigid body in-span. The center of mass of the body was on the neutral axis of the beam and within or outside the axial length of the body.

Wu and Chen [14] presented a modified lumped-mass transfer matrix method (LTMM) so that one may easily determine the natural frequencies and the corresponding mode shapes of a multi-step Timoshenko beam with various boundary conditions and carrying various concentrated elements with eccentricity of each lumped mass. Magrab [15] used the Laplace transformation method to obtain a solution for a Timoshenko beam mounted on elastic foundation with several combinations of discrete in-span attachments and with several combinations of attachments at the boundaries. The attachments include translation and torsion springs, masses and undamped single degree of freedom system. Recently, Lin and Wang [16] presented a method to determine the exact natural frequencies and mode shapes of hybrid beam composed of multiple elastic beam segments and multiple rigid bodies. Each rigid body connected with two adjacent elastic beam segments, has its own mass and rotary inertia and supported by a translational spring and/or rotational springs. Magrab [17] through his textbook, chapter 3 discussed the mathematical equations for Bernoulli–Euler beam with an in-span rigid extended mass. Graphical results for the variation in the modal frequencies and their associated modal shapes were presented in [17]. Very recently, the exact free vibration of multi-step Timoshenko beam system with several attachments including two degree of