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A local identification method for linear parameter-varying systems based on interpolation of state-space matrices and least-squares approximation

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ABSTRACT

This paper proposes a novel state-space matrix interpolation technique to generate linear parameter-varying (LPV) models starting from a set of local linear time-invariant (LTI) models estimated at fixed operating conditions. Since the state-space representation of LTI models is unique up to a similarity transformation, the state-space matrices need to be represented in a common state-space form. This is needed to avoid potentially large variations as a function of the scheduling parameters of the state-space matrices to be interpolated due to underlying similarity transformations, which might degrade the accuracy of the interpolation significantly. Underlying linear state coordinate transformations for a set of local LTI models are extracted by the computation of similarity transformation matrices by means of linear least-squares approximations. These matrices are then used to transform the local LTI state-space matrices into a form suitable to achieve accurate interpolation results. The proposed LPV modeling technique is validated by pertinent numerical results.

1. Introduction

The system identification framework of linear time-invariant (LTI) systems is well-established nowadays [1,2] and it has tackled a broad range of applications. However, different systems in a variety of applications can exhibit a time-variant behavior and the time-invariant condition is no longer satisfied [3,4]. For example, the impedance of a metal subjected to an electrochemical process [5] varies over time and the dynamics of the wings of a plane vary depending on the flight speed and flight altitude [6].

Therefore, enhanced models are needed to properly describe the behavior of these systems for which LTI models cannot be used. Linear parameter-varying (LPV) models allow describing the dynamics of linear systems whose behavior is affected by time-varying parameters called scheduling parameters [7,8] (e.g. the flight speed, the extension of an extendible robot arm, an external voltage source controlling an electronic circuit, etc.). The identification of LPV models is a very active research area [7–18]. Two main LPV identification approaches are commonly used in the literature: global [7,9,10,12,13] and local [8,11,14,16,17,19,20].

In a global approach, both the input and the scheduling signals are persistently exciting the system simultaneously in one

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global identification experiment. In a local approach, a set of local linear time-invariant (LTI) models is first identified using a set of local experiments and then interpolated to obtain an LPV model. The local experiments excite the system at different constant values of the scheduling parameters (different fixed operating conditions). LTI identification techniques and experiments are well-established, which makes the local LPV modeling approach attractive. However, since the local LTI models are identified at fixed scheduling operating conditions, LPV models based on a local approach are accurate if the scheduling parameters variations are slow with respect to the dynamics of the system [21]. Local LPV modeling cannot lead to LPV models able to accurately capture the system behavior related to the dynamic dependency on the scheduling parameters (dependency on time-derivatives of the scheduling parameters (continuous-time) or time-shifted versions of the scheduling parameters (discrete-time)). Local LPV models can only accurately identify systems with a static dependency on the scheduling parameters (dependency on the instantaneous time values of the scheduling parameters), because, by construction, the local LTI models identified at fixed scheduling operating conditions do not contain information about the dynamic dependency on the scheduling parameters. In this paper, we focus on local LPV modeling and we do not tackle the dynamic dependency on the scheduling parameters.

Interpolation of the state-space matrices of local LTI systems is often used in a local LPV approach [8,11,14,16,17,19]. The local LTI models are generated by separated identification steps. Since the state-space representation of LTI models is unique up to a similarity transformation, the variations of the state-space matrices of the local LTI models as a function of the scheduling parameters can be strongly affected by underlying similarity transformations. The state-space matrices of the local LTI models need to be represented in a common state-space form to somehow remove these additional degrees of freedom. This is needed to avoid potentially large variations as a function of the scheduling parameters of the state-space matrices to be interpolated due to underlying similarity transformations, which might degrade the accuracy of the interpolation significantly. In the literature, several approaches have been proposed to transform the local LTI models into a state-space representation suitable for interpolation. In [11], single-input single-output (SISO) systems are considered and the control canonical form is proposed as the state-space representation for all local LTI models. However, this form might suffer from numerical ill-conditioning for high-order systems [22]. In [14], an LPV approach based on balanced realizations of LTI models has been proposed. As discussed in [8,16], this method requires the manual sorting of the eigenvalues of the product of the Gramians of the local LTI models. This is a drawback since manual sorting requires intuition, experience and interaction from the user. In [8,16], an LPV approach based on a series connection of low-order state-space submodels related to the poles and zeros of the local LTI models has been proposed. A drawback of this technique is that the poles and zeros of the local LTI models need to be manually sorted. Recently, the use of observability or controllability matrices has been proposed in [19] to compute similarity transformation matrices and transform the local LTI models before performing an interpolation step. The computation of observability or controllability matrices might again suffer from numerical ill-conditioning for high-order systems.

This paper proposes a novel state-space interpolation technique to generate LPV models starting from a set of local LTI models. The proposed approach avoids the computation of canonical forms, observability or controllability matrices, balanced realizations and manual sorting of specific quantities (such as poles and zeros). Underlying linear state coordinate transformations for a set of local LTI models are extracted by the computation of similarity transformation matrices by means of linear least-squares approximations. We propose two novel least-squares-based algorithms using fixed and dynamic reference state vector trajectories for the computation of the similarity transformation matrices. These matrices are then used to transform the LTI models into a suitable state-space representation which allows obtaining accurate interpolation results. The proposed technique is easy-to-implement and relies on the use of robust numerical tools.

The paper is organized as follows. Section 2 describes the proposed LPV modeling approach. Pertinent numerical results illustrate and validate the proposed technique in Section 3. The conclusions are presented in Section 4.

2. LPV modeling

2.1. Background

The LPV system class can be seen as an extension of LTI systems, where the signal relations are considered to be linear, but the model parameters are assumed to be functions of time-varying signals called scheduling parameters $\mathbf{p}(t) \in \mathbb{R}^{M\times 1}$ that assume values in a bounded scheduling space $\mathcal{P}$. An LPV model in the form:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{D}(\mathbf{p}(t))\mathbf{u}(t) \tag{1}$$

with $\mathbf{A}(\mathbf{p}(t)) \in \mathbb{R}^{n_x\times n_x}$, $\mathbf{B}(\mathbf{p}(t)) \in \mathbb{R}^{n_x\times n_u}$, $\mathbf{C}(\mathbf{p}(t)) \in \mathbb{R}^{n_y\times n_x}$, $\mathbf{D}(\mathbf{p}(t)) \in \mathbb{R}^{n_y\times n_u}$ is computed by the proposed modeling method. In what follows, we omit the time dependence of the scheduling parameters for ease of notation. Let us consider a set of K local experiments that excite the system under study at different constant values $\mathbf{p}_k$, $k = 1, \ldots, K$ of the scheduling parameters (different fixed operating conditions). Based on these data sets, a set of local LTI models $\mathbf{A}(\mathbf{p}_k)$, $\mathbf{B}(\mathbf{p}_k)$, $\mathbf{C}(\mathbf{p}_k)$, $\mathbf{D}(\mathbf{p}_k)$ can be

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