Signal Processing 144 (2018) 7–18

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Extraction of buried multidimensional signals and images in mixed sources of noise

Nourédine Yahya Bey

Université François Rabelais, Parc de Grandmont, F-37200 Tours, France

ARTICLE INFO

Article history:
Received 30 November 2016
Revised 22 August 2017
Accepted 19 September 2017
Available online 22 September 2017

Keywords:
Denoising
Buried signals
Buried images
White noise
Colored noise
Shot noise
Multi-dimensional Fourier analysis
Wavelet denoising
Image denoising
Image restoration

ABSTRACT

In this paper, multi-dimensional extension and additional properties of already proposed extraction methods of buried one-dimensional signals in noise are developed. It is shown that heavy denoising uses no a-priori information, works without averaging or smoothing in the time or frequency domain with computation times much lower than those needed by ensemble averaging operations. Extraction is achieved independently of the nature of noise and locations of its spectral extent. Heavy denoising performances, comparative results with wavelets and other denoising algorithms, are illustrated via buried two-dimensional signals and images in noise. Proposed restoration of buried images in mixed sources of noise is able to preserve image information carried by fine structure, edges and texture. This ability opens novel perspectives for image restoration.

1. Introduction

Heavy denoising or extraction from noise of multidimensional signals has significant applications in many areas of different sciences (see, e.g., [1–10]). Particularly, buried two-dimensional signals in image processing remains to date an important challenge since images are often corrupted by various natures of noise in different systems as, for example, acquisition and transmission devices. They also can be taken in poor conditions. Much work is devoted to the subject and no attempt is made in our reference section to present an exhaustive list of various proposed denoising algorithms and their corresponding merits (see, e.g., [11–15]).

The task of image denoising is to reduce noise while preserving image information. Most of image denoising algorithms are based on the model of noise and all of them uses a global or local generic image smoothness [11]. This last point represents the principal drawback of all existing algorithms (see, for example, various 2D wavelet denoising algorithms with their thresholding and shrinking rules together with filtering methods [16,17]) since no difference is made between noise and small buried details in the noisy image. Elimination of noise together with these small details creates various distortions indexed in lists of denoising artifacts. One notes that many methods [18] have been investigated in pertinent literature and one finds that if these algorithms performs very well when their intrinsic hypothesis are met, their failed in many real-world situations due to their level of sophistication, feature settings and specific models. Moreover, to the best of our knowledge (see, e.g., [19–21]), any of the proposed algorithms extracts buried details (fine structure, edges and texture) in noisy images independently of the nature of noise and extension of its spectral support.

We proposed in [22] two extraction methods applied to buried “one-dimensional” signals that work independently of the nature of noise (white or colored, Gaussian or not) and its spectral support. This heavy denoising or extraction from noise does not use a-priori information neither on the signal to be extracted nor on the nature of noise. No averaging or smoothing in the direct time or dual frequency domain is performed. Moreover, computation times of the two proposed denoising methods are much lower than those needed by ensemble averaging operations to achieve comparable variance reduction of noisy spectral estimates. It is crucial to notice that denoising methods extract *clean* power spectral estimates of buried signals in noise as briefly summarized hereafter:

1) The first method, called MFED (Modified Frequency Extent Denoising), works in invariant observation space (one realization). It is based on increasing the sampling frequency of

E-mail address: nouredine.yahyabey@phys.univ-tours.fr

https://doi.org/10.1016/j.sigpro.2017.09.022
0165-1684/

continuous-time signals, transforming and windowing obtained spectra. Covariance of white noisy spectral estimates (Gaussian or not) decreases as the square of over-sampling factors whereas covariance of colored noise spectral estimates vanishes within the signal bandwidth.

2) The second method, called CFED (Constant Frequency Extent Denoising), works in modified observation space (collection of realizations). Here the signal-to-noise ratio is increased by increasing paving in the observation space, transforming and applying decimation to obtained sample spectrum. Covariance of white and/or colored noisy spectral estimates (Gaussian or not) is reduced proportionally to collected processes.

Application procedures, performances of proposed methods and comparative results with other denoising techniques (modified periodogram method, bispectrum estimation and wavelet denoising) are reported in [23].

In this paper, proposed extension of above results to multi-dimensional buried signals and images is motivated by,

1) citations of one-dimensional observations [22,23] in pertinent literature (see, e.g., [24–27]),
2) the necessity to provide additional properties demonstrating ability of denoising methods to eliminate cross-terms appearing in power spectral density computations,
3) the opportunity to propose the theoretical frame of image/video denoising,
4) implementation of a novel restoration method of buried images in mixed sources of noise.

Section 2 specifies some definitions and notations suitable to multi-dimensional extension, whereas Sections 3 and 4, use these notations to develop multi-dimensional MFED and CFED with above mentioned additional properties. For the sake of illustration, extraction performances of buried two-dimensional signals and images are discussed in Section 5. A novel restoration method of buried images in mixed sources of noise is implemented and its performance compared to most used algorithms. In order to avoid a lengthy report, video denoising with its specific notations, definitions and concepts can be addressed in a forthcoming work.

2. Definitions and notations

2.1. Signal representation

Let $\Gamma(\mathbf{f})$ be the $\mathcal{N}$th dimensional bandpass spectrum of the zero-mean real $\mathcal{N}$th dimensional signal $s(\mathbf{t})$ defined by,

$$\Gamma(\mathbf{f}) = 0, \qquad (\mathbf{f})_{\mathbf{min}} \geq |\mathbf{f}| \geq (\mathbf{f})_{\mathbf{max}}, \tag{1}$$

where $\forall k = 1, 2, \cdots, \mathcal{N}$, $(f_k)_{\min} \geq |f_k| \geq (f_k)_{\max}$ are bounds of spectral supports of $\Gamma(f_k)$.

Let us add to $s(\mathbf{t})$, an $\mathcal{N}$th dimensional zero-mean wide-sense real stationary noise $b(\mathbf{t})$ to form the process,

$$z(\mathbf{t}) = s(\mathbf{t}) + b(\mathbf{t}) \,. \tag{2}$$

Finite observation of $z(\mathbf{t})$ in the interval **[0, T]** available at the output of low-pass filters of cut-off frequencies $(\mathbf{f})_{\max}$ yields,

$$z_{\mathbf{T}}(\mathbf{t}) = \begin{cases} b_{\mathbf{T}}(\mathbf{t}) + s_{\mathbf{T}}(\mathbf{t}), & \mathbf{t} \in [\mathbf{0}, \mathbf{T}] \\ 0, & \text{otherwise}\,. \end{cases} \tag{3}$$

Observation intervals are so that $\mathbf{T} = [T_1, \cdots, T_k, \cdots, T_{\mathcal{N}}]$ where $T_k (f_k)_{\max} \gg 1$.

2.2. Multi-dimensional sample power spectral density (SPSD)

By considering the instants $\mathbf{t_n} = \mathbf{n}/\mathbf{f_e}$ where $\mathbf{f_e} \geq \mathbf{2}(\mathbf{f})_{\max}$ represent sampling frequencies with $\mathbf{N} = \mathbf{T}\mathbf{f_e}$, we can define the $\mathcal{N}$th dimensional discrete-time process $z(\mathbf{n})$. Given $z(\mathbf{n})$, we form the estimate,

$$\Phi(\mathbf{f}, \mathbf{f_e}, \mathbf{T}) = \left| \frac{1}{\prod_{k=1}^{\mathcal{N}} T_k} \mathrm{DFT}_{[\mathcal{N}D]}(z(\mathbf{n})) \right|^2, \tag{4}$$

where $\mathbf{f}$ and $\mathbf{f_e}$ are defined above. The subscript in $\mathrm{DFT}_{[\mathcal{N}D]}(z(\mathbf{n}))$ denotes the $\mathcal{N}$th dimensional Discrete Fourier Transform of $z(\mathbf{n})$.

For the sake of clarity, we let the estimate $\Phi(\mathbf{f}, \mathbf{f_e}, \mathbf{T})$ depending explicitly on : $\mathbf{f}$, $\mathbf{f_e}$ and $\mathbf{T}$. This estimate is evaluated at $\mathbf{f} = \mathbf{m}/\mathbf{T} = [m_1/T_1, \cdots, m_k/T_k, \cdots, m_{\mathcal{N}}/T_{\mathcal{N}}]$ where $m_k = 0, \cdots, N_k - 1$ and $k = 1, \cdots, \mathcal{N}$.

Since *no averaging* is performed in the time or frequency domain then (4) defines multidimensional Sample Power Spectral Density "($\mathcal{N}$−SPSD)" of the process $z(\mathbf{n})$.

3. Multi-dimensional MFED

Here, denoising procedure extends analysis frequency ranges of *one realization* of the multi-dimensional process observed in $[\mathbf{0}, \mathbf{T}]$. This is termed "Modified Frequency Extent Denoising". We derive hereafter spectral representations $\Gamma(\mathbf{f}, \mathbf{f_e}, \mathbf{T})$ of the signal and $\Psi(\mathbf{f}, \mathbf{f_e}, \mathbf{T})$ of noise as a function of over-sampling factors $\mathbf{B}$. For easy reference, we recall pertinent results of [22] as computations proceed.

3.1. Spectral representation of noise

Let us represent $\Psi(\mathbf{f}, \mathbf{f_e}, \mathbf{T})$ by the $\mathcal{N}$th dimensional matrix of coefficients,

$$\mathrm{L} = \{c_{\mathbf{0}}, c_{\mathbf{1}}, \cdots, c_{\mathbf{N-1}}\} \,. \tag{5}$$

We recall that for $\mathcal{N}$=1 (one-dimensional case, p.2466 of [22]), $L = \{c_0, \cdots, c_{N-1}\}$.

Now, given $\mathbf{B} = [B_1, B_2, \cdots, B_{\mathcal{N}}]$, we over-sample $z(\mathbf{t})$ with $\mathbf{B}\mathbf{f_e}$. Obtained

$$S = \{r_{\mathbf{0}}, r_{\mathbf{1}}, \cdots, r_{\mathbf{BN-1}}\} \,. \tag{6}$$

For $\mathcal{N}$=1 (one-dimensional case), S reduces to $S = \{r_0, \cdots, r_{B(N-1)}\}$ and it is then decomposed into N sub-sequences $\{S_0, \cdots, S_{N-1}\}$, Here also, the $\mathcal{N}$th dimensional matrix, S, as given by (6), can be decomposed into **N** sub-matrixes as follows,

$$S = [S_{\mathbf{0}}, \cdots, S_{\mathbf{N-1}}], \tag{7}$$

where explicit writing of, S, as given by (7), yields,

$$\begin{aligned} S = [&S_{0\cdots0}, S_{0\cdots1}, \cdots, S_{0\cdots0B_1(N_1-1)}; \\ &S_{0\cdots0\,B_1(N_1-1)}, S_{0\cdots1\,B_1(N_1-1)}, \cdots, S_{0\cdots B_2(N_2-1)B_1(N_1-1)}; \cdots; \\ &S_{0B_{\mathcal{N}-1}(N_{\mathcal{N}-1}-1)\cdots B_1(N_1-1)}, S_{1N_{\mathcal{N}-1}(B_{\mathcal{N}-1}-1)\cdots B_1(N_1-1)}, \cdots, \\ &S_{B_{\mathcal{N}}(N_{\mathcal{N}}-1)N_{\mathcal{N}-1}(B_{\mathcal{N}-1}-1)\cdots B_2(N_2-1)B_1(N_1-1)}], \end{aligned}$$

Each sub-matrix, $S_{\mathbf{m}}$, has the set of coefficients,

$$S_{\mathbf{m}} = \{r_{\mathbf{mB}}, r_{\mathbf{mB+1}}, \cdots, r_{(\mathbf{m+1})\mathbf{B-1}}\},$$

where $\mathbf{m} = [m_1, m_2, \cdots, m_{\mathcal{N}}]$ and $m_k = 0, 1, \cdots, N_k - 1$ for $k = 1, 2, \cdots, \mathcal{N}$. Now, sub-spectra written as a function of coefficients $r_{\mathbf{mB}}$ yield,

$$S_{\mathbf{m}} = \sum_{\mathbf{p=0}}^{\mathbf{B-1}} r_{\mathbf{mB+p}} \delta[\mathbf{f} - (\mathbf{Bm} + \mathbf{p})/\mathbf{T}], \tag{8}$$

where $\delta[\mathbf{f}]$ is the $\mathcal{N}$th dimensional unit impulse function.

On the other hand, the matrix L, as given by (5), can similarly be decomposed into **N** sub-matrixes,

$$\mathrm{L} = [L_{\mathbf{m}}],$$

where,