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Orthogonal circulant structure and chaotic phase modulation based analog to information conversion

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ABSTRACT

Modulated wideband converter (MWC) is used to implement analog to information conversion (AIC) to realize sub-Nyquist sampling of sparse multiband analog signal. In this paper, we combine orthogonal circulant matrix with chaos to develop an orthogonal circulant structure and chaotic phase modulation based modulated wideband converter (OCSCPM-MWC). Firstly, random waveforms of different channels in the system correspond to the row vectors of measurement matrix. We use the frequency-domain approach to construct orthogonal circulant matrix. The waveforms of different channels can be generated by various cyclic shifts of the same original waveform and orthogonal to each other. This can greatly reduce the degrees of freedom of the random waveforms, and a superior recovery performance can be obtained. Secondly, we construct chaotic phases of generating elements of orthogonal circulant matrix. The input signal is modulated according to the power spectrum of chaos. We derive the theoretical condition for the orthogonal circulant measurement matrix to satisfy expected restricted isometry property (ExRIP). In addition, we analyze the impact of measurement noise and signal noise on the OCSCPM-MWC system, respectively. Computer simulations are carried out under different situations and they confirm the effectiveness of the proposed approach.

1. Introduction

In the field of wireless communication, baseband signals are always modulated by high carrier frequencies through radio frequency (RF) technology. Radio signal always consists of a small number of narrowband transmissions across a wide spectrum range. Sparse multiband model is a convenient way to describe such signal. It is prohibitive to sample sparse multiband signal under the Nyquist rate due to the wide spectrum range. Many efforts have focused on pursuing an efficient sampling way with sub-Nyquist rate by taking advantages of its sparse structure. When the carrier frequencies are known, Landau proposes to demodulate the signal by its carrier frequencies following with a low-pass filter and sample at the minimal rate that is equal to the efficient spectrum occupation [1]. In addition, periodical non-uniform sampling [2] is used to sample band-pass signal directly. In practical applications, the actual carrier frequencies are always unknown and dynamically varying. For such case, multicoset sampling strategy is developed to sample the multiband signal at a low rate [3,4]. However, there are two difficulties for implementing multicoset sampling in practice. First, the attempt to acquire a radio signal with practical analog-to-digital converters (ADCs) results in a loss of the spectral contents beyond the analog bandwidth of state-of-the-art devices. Second, maintaining accurate time shifts between the ADCs in the order of the Nyquist interval is difficult for implementation.

The theory of compressed sensing/compressive sampling (CS) [5–9] was brought to the research forefront by Donoho, Candès, Romberg and Tao. It provides an efficient framework for acquisition and processing of sparse signal. The core of CS is that a sparse or compressible signal can be exactly recovered from non-adaptive and incomplete linear measurements by a convex optimization algorithm with a high probability. The original process of CS can be considered as an analog signal being uniformly sampled at Nyquist rate and then multiplied by a measurement matrix to obtain the compressive results. To reduce the sampling rate, a practical approach for direct sampling and compression of analog signal is analog-to-information converter (AIC). For sparse multiband signals, AIC can reduce sampling rate simply depending on the useful information (or sparsity level) embedded in the signal.

The random sampling based AIC is proposed by Baraniuk et al. [10,11]. It enables sub-Nyquist acquisition and processing of wide-

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band signals that are sparse in a local Fourier representation by building a Random Analog to Digital Converter (RADC). Random demodulator (RD) is developed in [12–14], which uses modulation, filtering, and sampling to produce a low-rate set of compressive measurements. Beyond that, Tropp also proposes random filtering as a new paradigm for compressive signal acquisition [15]. The research team of Eldar proposes the modulated wideband converter (MWC) [16] aims to solve the challenging problem of spectrum-blind sub-Nyquist sampling and recovery of multiband signals, in which the carrier frequencies are unknown to the receiver, or vary in time. The system consists of multiple sets of multipliers, low pass filters and low speed of A/D converters. At the first glance, the sampling and compression techniques used in MWC and RD look similar. A comprehensive comparison is investigated in [17] for these two sub-Nyquist acquisition strategies in terms of three metrics: robustness to model mismatch, required hardware accuracy, and software complexities. The most noticeable difference is the discrete multitone setting of RD in contrast to the analog sparse multiband model of MWC. Besides, MWC are more flexible in terms of hardware implementation and computationally efficient. Beyond that, segmented compressed sampling based AIC is proposed in [18] with analog signal is first segmented into multiple segments and then measured by a number of parallel branches of mixers and integrators (BMIs).

In addition to the directly sub-Nyquist sampling of sparse multiband signals, AIC also has many applications in other various domains. For example, in cognitive radio (CR) settings, the terminals need to monitor the spectrum usage among different bands at a given time and dynamically access the available spectrum. Due to the plenty of temporarily unused spectrum in wideband CR networks, AIC is motivated for wideband spectrum sensing under sub-Nyquist rate with improved efficiency and low complexity [19–22]. Also, AIC has been used in Wireless Sensor Networks (WSNs), which comes from the fact that the CS framework lends itself to the accurate recovery of large and distributed signals through the collection of a small number of samples. This technique is able to reduce global scale communication cost and the power consumption, resulting in an extending of the lifetime of sensor Networks [23].

Until now, many initiatives focus on hardware implementation and applications of AIC. However, random mixing waveforms of AIC are independent of each other with high degrees of freedom and there are too many elements need to be stored. In addition, due to the mixing waveforms in different channels need to be different so as to capture linearly independent mixtures of the spectrum, the hardware size of MWC may be too large to fit into a mobile CR device for wideband spectrum sensing [24]. To address this problem, Romberg proposes circulant convolution based CS [25] and proves theoretically that the circulant convolution measurement matrix has restricted isometry property (RIP) [6] with a high probability. After that, some researchers focus on the applications of circulant matrix in AIC. For example, circulant measurement matrices based on deterministic sequences, particularly m-sequence, Legendre sequence, and complementary sequences etc. are investigated in [26–28]. While these kind of deterministic sequences can reduce hardware complexity, they only exit for a fixed period length and are not universal for practical applications. Besides, the circulant matrix can also be constructed by Zadoff-Chu sequence or other special sequence in frequency domain [26,29,30]. In addition, sparse circulant matrices developed in [31], whose generating sequence is a sparse sequence, could keep the energy balance of subspaces and have similar noise robustness with deterministic circulant matrices. There are also some other researches focus on reducing the number of physical parallel channels in MWC from m to 1 [32,33] at the cost of a longer processing time.

In this paper, we propose a new structured design of AIC for the sub-Nyquist sampling of sparse multiband signal. Our main objective is not only to reduce the degrees of freedom of random waveforms, but also with a good recovery performance.

The first main contribution of our work is the novel orthogonal circulant structure and chaotic phase modulation based modulated wideband converter (OCSCPM-MWC). The key idea is based on two innovations: (1) From the implementation perspective, it is desirable that the measurement matrix requires the minimal memory for storage and lowest computational cost for recovery. Random waveforms of different channels in the system correspond to the row vectors of measurement matrix. We construct orthogonal circulant matrix in the frequency domain. Thus, the waveforms of different channels can be generated by various cyclic shifts of the same original waveform that are orthogonal to each other. Compared with fully random matrix (Gaussian or Bernoulli matrix), this can greatly reduce the degrees of freedom. Meanwhile, orthogonal circulant matrix can be implemented with two FFTs, which is computationally efficient. (2) The input signal multiplied by a circulant matrix can be defined as $\mathbf{C}\cdot\mathbf{x}=\frac{1}{M}\mathbf{F}^H\mathrm{diag}(\sigma)\mathbf{F}\cdot\mathbf{x}$ (refer to (21) in details). This process can be implemented by first transforming the signal to the frequency domain by FFT. Generally, phases encode the signal location information in spectrum. Since such location is unknown in the sensing process, we choose to generate chaotic phases of the diagonal vector σ so as to modulate the input signal by taking advantage of sharp autocorrelation function and wideband power spectrum property of chaos.

The second contribution of this paper is the theoretical analysis of OCSCPM-MWC. It consists of three parts. (1) For the proposed OCSCPM-MWC system, we demonstrate that the number of measurements required for both uniform and non-uniform recovery are optimal. (2) Moreover, we derive the theoretical condition that guarantees OCSCPM-MWC to satisfy expected restricted isometry property (ExRIP) [34]. (3) We also analyze the impact of both measurement noise and signal noise that is present in the signal before acquisition in OCSCPM-MWC system, respectively.

This paper is organized as follows. Section 2 briefly presents the theory of MWC system. In Section. 3, we focus on the construction of OCSCPM-MWC, including the design of sampling structure of OCSCPM-MWC system and the construction of orthogonal circulant matrix based on chaotic phase modulation. Section. 4 is devoted to the theoretical analysis of OCSCPM-MWC, which consist of RIP and RIPless property of OCSCPM-MWC measurement matrix, theoretical condition for OCSCPM-MWC to satisfy ExRIP, the impact of measurement noise and signal noise on OCSCPM-MWC system. Numerical simulation results are presented in Section. 5. Concluding remarks are given in Section. 6.

2. Review of MWC system

2.1. Sparse multiband model

Let x(t) be a real-valued analog signal, its Fourier transform is defined as

$$X(f)=\int_{-\infty}^{\infty}x(t)e^{-j2\pi ft}dt \tag{1}$$

The spectrum is assumed to be bandlimited with $\mathcal{F}=[-1/2T,\ 1/2T)$. We denote $f_{NYQ}=1/T$ as the Nyquist rate of $x(t)$. The set of sparse multiband signals can be defined as the set containing all signals $x(t)$, such that the support of the Fourier transform $X(f)$ is contained within a union of N disjoint bands in $\mathcal{F}$, and each bandwidth does not exceed B. The spectrum of sparse multiband signals is shown in Fig. 1.