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# Low rank constraint and spatial spectral total variation for hyperspectral image mixed denoising

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ABSTRACT

Due to the environmental and instrumental factors, the hyperspectral image (HSI) is corrupted by various noise, which inevitably affects the subsequent HSI-based applications. Several band by band Total Variation(TV)-regularized low rank based models have been proposed for HSI mixed denoising. However, these methods only utilize the spatial smooth constraint in a separated way, but ignore the local spectral smooth property, which may cause the undesirable jagged spectral distortion. To cope with this problem, we propose a novel low rank constraint and spatial spectral total variation regularization model. First, we adopt the weighted nuclear norm to restore the clean HSI from the mixed noise based on the low rank property. Then, the spatial spectral total variation is modeled as a special regularization to further remove the Gaussian noise and enhance the local spatial and spectral smoothness. Finally, an iterative strategy based on the Alternating Direction Method of Multipliers is designed to solve the derived optimization problem. Extensive experiments demonstrate the superiority of the proposed model in terms of mean PSNR, mean SSIM, mean spectral angle distance and visual quality. Especially, the proposed model is very effective for suppressing the jagged spectral distortion while removing the noise.



## 1. Introduction

With the ability of sensing the electromagnetic signals in a very high spectral resolution, the hyperspectral imagery has drawn many attentions from various fields [1], including the urban planning, vegetation mapping, and monitoring. Nevertheless, the hyperspectral image (HSI) obtained by the sensors still suffers from various noise sources, e.g., the atmospheric haze [2] and instrument noise [3], which severely degrade the quality of images and further affect the subsequent HSI-based applications, such as compression [4,5], unmixing [6], classification [7,8], and target detection [9–11]. To cope with these problems, an advisable strategy is to do the noise removal before the subsequent applications.

Due to the fluctuations in power supply, thermally induced dark current, and atmospheric absorption, the Gaussian noise is usually visually perceptible [12–14]. The salt and pepper noise usually arises when some of the sensor is saturated or fails to sample [15,16]. For the hyperspectral scanner using either the whiskbroom technology (e.g., the airborne HyMap [17]) or the pushbroom technology (e.g., the space borne EO-1 Hyperion [18]), the stripes are visually perceptible due to the miscalibration [12,19]. More seriously, if entire rows or columns are missing owing to the physics damage of the scanner, the resulting HSIs will be corrupted by obvious deadline noise [20]. Affected by various noise sources, the obtained HSIs usually suffers from the mixture of different noise in the real application. Within this paper, we mainly focus on the HSI mixed denoising, which means the removal of mixture of different noise, including the Gaussian noise, salt and pepper noise, and deadline noise.

Until now, many different methods have been proposed for HSI mixed noise removal. In [21], Letexier et al. proposed a multidimensional wiener filtering (MWF) by modeling the HSI as a 3-D tensor and removing the noise with tensor analysis method. In [22], a spectral-spatial adaptive total variation (SSAHTV) was proposed, in which both the spectral and spatial noise differences are taken into consideration for noise reduction. However, the SSAHTV is insensitive to the image details and may blur the edges. To overcome this problem, a spectral-spatial kernel regularization was proposed in [23] to maintain both the spectral correlations and spatial structure information. Meanwhile, a similar strategy based on the anisotropic spectral-spatial total variation was proposed in [24], in which the anisotropic total variation was used to enhance smoothness in both spectral and spatial dimension. In [25], a spatial-spectral view fusion strategy was proposed for HSI denoising, in which the total variation was applied to both the spatial

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and spectral views. Moreover, some volumetric denoising methods have been proposed and can be adopted for HSI mixed noise removal, such as BM4D [26], which is based on the nonlocal similarity and wiener filtering.

Recently, several low rank based methods have been proposed for HSI mixed denoising. Suppose the clean HSI is $\mathbf{L}^{M_x\times M_y\times M_z}$, rearrange each spectral band into a vector and combine them into the Casorati matrix $\mathbf{L}^{M_xM_y\times M_z}$. Then the low rank property of HSI can be explained in the following two aspects. (1) Based on the linear mixing model (LMM), the HSI is actually composed of finite number of pure endmembers [1]. That is, the rows in Casorati matrix $\mathbf{L}^{M_xM_y\times M_z}$ are quite interrelated. In other words, the matrix $\mathbf{L}^{M_xM_y\times M_z}$ has the low row rank property. (2) As is known, the HSI is spectrally smooth, which means that the values in neighboring wavelengths are highly correlated. So the columns in matrix $\mathbf{L}^{M_xM_y\times M_z}$ are quite correlated, which indicates that the matrix $\mathbf{L}^{M_xM_y\times M_z}$ has the low column rank property.

In [27], the low rank matrix recovery (LRMR) model was first proposed by Wright et al., which aims at separating the low rank matrix $\mathbf{L}$ from the corrupted observation $\mathbf{D}$, where $\mathbf{D}=\mathbf{L}+\mathbf{S}$, and $\mathbf{S}$ is the unknown sparse noise. Based on the reasonable low rank assumption, Zhang et al. [20] adopted the LRMR to recover the clean HSI $\mathbf{L}$ from the corrupted dataset $\mathbf{D}$, where $\mathbf{D}=\mathbf{L}+\mathbf{S}+\mathbf{N}$, and $\mathbf{N}$ is the unknown Gaussian noise. Despite the impressive denoising performance of the LRMR, the Achilles' heel is the absence of spatial constraint. To cope with the above problem, a two-phase matrix decomposition scheme was proposed in [28] by employing the LRMR in the first phase for a preliminary decomposition and then using the band by band TV regularization for a spatial compensation in the second phase. Similarly, in [29], a low rank spectral nonlocal approach (LRSNL) by utilizing the LRMR for precleaning and then adopting the spectral nonlocal (SNL) method was proposed. To further improve the denoising performance of low rank based methods, a novel TV-regularized low rank matrix factorization (LRTV) model was proposed in [30], which combines the low rank matrix factorization with the band by band TV regularization to utilize the spectral and spatial information simultaneously. To go a step further, in our previous work [31], we proposed a novel HSI denoising model TWNNM based on the weighted nuclear norm minimization and the band by band TV regularization.

Despite the superior denoising performances of LRTV and TWNNM, these low rank constraint and band by band TV-regularized methods still suffer from some weaknesses. (1) As is known, the HSI is spatially piece-wise smooth and spectrally smooth [1]. However, both LRTV and TWNNM can only utilize the spatial smooth constraint but ignore the spectral smooth constraint. Consequently, these band by band TV-regularized methods may easily cause some jagged distortions in the spectral domain (please refer to Section 2.2 for a more detailed description). (2) Another drawback of the spatial TV-based methods is the undesirable oil painting effect.

To exploit the global gradient information from all the spectral bands simultaneously, in our previous paper [32], we proposed a HSI denoising model STWNNM by utilizing both the low rank constraint and Structure tensor Total Variation (STV) [33]. Within the STWNNM, the STV is adopted to utilize the gradient information from all spectral bands, and the non-negative, rotationally symmetric convolution kernel is used to exploit the local spatial information. The STWNNM can effectively remove the mixed noise, especially when the noise is with obvious structure (e.g., the deadline noise with the width of 3 pixels or even more). Nevertheless, as mentioned in [32], the STWNNM uses a fixed parameter to regularize all the bands simultaneously, which cannot guarantee the optimal values of all bands simultaneously. What's worse, similar to the LRTV and TWNNM, the STWNNM also ignores the local spectral smooth property of HSI, which may easily cause some spectral distortions.

On the other hand, some researchers have tried to exploit more information from the spectral derivative domain for HSI denoising and classification. A wavelet based HSI denoising model was proposed in [2], which was actually based on the prior knowledge that the noise level is elevated in the spectral derivative domain, and that the signal regularity in the spectral and spatial domains are quite dissimilar. In [34], Tsai and Philpot adopted the derivative operation to capture more important spectral details for landcover classification. Meanwhile, a novel classification model based on the combination of magnitude, derivate and shape features was proposed in [35]. All these methods further indicate the effectiveness of spectral derivate information for denoising/classification and motivate us to carry on a further research.

More recently, a novel HSI denoising model termed as the spatial spectral total variation (SSTV) was proposed in [14] by Aggarwal and Majumdar, which extends the traditional spatial TV into three dimensions to utilize both the local spatial and spectral information. The SSTV model can remove mixed Gaussian and sparse noise from HSI, and have got a superior performance than many other noise removal methods, e.g., the LRMR [28], PCAW [36], and GSP [37]. Nevertheless, there are still two main problems within the SSTV: (1) The paper [14] only uses the SSTV within the *maximum a posteriori* (MAP) denoising framework and can only capture the local spatial and spectral information, but ignores the global low rank property of HSI. Many works indicate that the low rank property of HSI could be utilized to further exploit the essential property of HSI and improve the final performance, for example, the 3D hyperspectral images analysis [38], hyperspectral denoising [39], and hyperspectral destriping [40]. (2) In [14], there is a lack of detailed explanation about why the spatio-spectral TV works well for HSI. To cope with the above problems, we propose a novel HSI mixed denoising model, termed as Low rank constraint and spatial spectral total variation (LSSTV). Different from the SSTV and other HSI denoising models, we first model the HSI mixed denoising as the weighted nuclear norm minimization (WNNM) problem to utilize the global low rank property of HSI, and then adopt the spatial spectral total variation as a special regularization term to enforce the spatial and spectral smoothness of the restored HSI. Besides, within this paper, we add a simple and explicable explanation about the jagged distortion caused by the band by band TV, and explain the reason why LSSTV could relieve this undesirable jagged distortion while removing the noise.

Compared with the existing literatures, the main contributions of our work can be summarized as follows.

(1) We propose a novel low rank and regularization based model for HSI mixed denoising, which is able to better utilize the global low rank property and the local spatial and spectral smooth properties of HSI, and hence facilitates the whole denoising process. In the proposed LSSTV model, the weighted nuclear norm minimization model is used to separate the clean HSI from the mixed noise based on the low rank property of HSI, and the spatial spectral total variation is utilized to further remove the Gaussian noise and enhance the local spatial and spectral smoothness.

(2) The global low rank constraint and local spatial spectral constraints are modeled into a uniform framework in LSSTV, and the effects between them can promote each other. The iterations based on the Alternating Direction Method of Multipliers (ADMM)[41] are carried out to solve the LSSTV effectively. Both the theoretic analysis and experimental results confirm the convergence and denoising performance of the proposed LSSTV. Especially, the LSSTV turned out to be very effective for sup-