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Spectrum optimization via FFT-based conjugate gradient method for unimodular sequence design

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ABSTRACT

In this paper, we propose a unified framework for unimodular sequence design with different uses. We achieve the task by minimizing the residual error between the designed power spectrum density (PSD) and the target one. This unified metric includes the objective functions of PSD fitting, spectral mask reduction, improving signal-to-interference-plus-noise ratio (SINR), minimizing integrated sidelobe level (ISL), complementary set of sequences (CSS) design, and orthogonal set of sequences (OSS) design as special cases. We solve the PSD residual error minimization using conjugate gradient (CG) method, which enjoys reliable local convergence and good convergence rate. We derive a way to employ fast Fourier transform (FFT) in calculating the gradient with respect to the sequence's phase, so that the CG method can be implemented efficiently. Due to the inherent nature of gradient method, the proposed method is very flexible and it can tackle the designs with composite objectives, which are more challenging and often intractable for the existing methods. Comparisons with the state-of-the-art methods indicate that the proposed method can achieve better or equally good results with much reduced execution time.

1. Introduction

Probing waveform design is a hot topic of current interests due to its ubiquitous applicability in active sensing (including radar, sonar, and communications) [1–3]. An active sensing system gathers and determines useful information of targets and propagation medium via transmitting and receipting one or more chosen waveforms. So it is not surprising that the probing waveforms directly affect the performance of the active sensing system. With the ongoing improvement of digital technology, promotion on computation capacity, as well as the emerging autonomous decision-making ability, it is recognized that the prospective performance enhancement and functionality breadth of probing waveform is tremendous.

In waveform design, the ambiguity function (AF) plays an important role and it is the principal tool for assessing the resolution, accuracy and ambiguity of the target range and radial velocity measurements. In the past decades, extensive efforts have been devoted to seeking waveforms with good autocorrelation function (ACF, i.e. the zero Doppler cut of the AF) [4–19]. Various metrics have been put forward to measure the goodness of a waveform's ACF sidelobes, including integrated sidelobe level (ISL), peak sidelobe level (PSL), and their variants. As the ACF and the power spectrum density (PSD) form a Fourier transform pair, most of these methods are optimizing the PSD implicitly. For example, methods are trying to flatten the PSD to reduce ISL in discrete sequence design [4–7,13]. While in deigning continuous-time waveform, e.g. nonlinear frequency modulations (NLFMs) [14–16] or polyphase-coded FMs (PCFMs) [17–19], properly tapering the PSD is applied to suppress the range sidelobes. Recently, optimization of the full AF behavior is also gaining focus. For example, AF shaping is performed in [20,21] to suppress the strong unwanted returns from certain range-Doppler bins foreseen. To better distinguish targets close in range and velocity, studies [22,23] suppress a small area of the AF that is close to the mainlobe.

Sequence sets that feature desired correlation properties are also playing an important role in many emerging applications, e.g. multi-input multi-output (MIMO) radar [24,25] and noncoherent pulse compression (NCPC) [26,27]. MIMO radar systems can obtain a greatly increased virtual aperture compared to traditional phased-array radar when it transmits orthogonal set of sequences (OSS) [28–31]. The OSS is required to possess small auto- and cross-correlation magnitude to maintain the pulse compression performance and minimize the cross-interference. In a NCPC system, further sidelobe reduction can be achieved using complementary set of sequences (CSS) [27,31,32], whose autocorrelation is given by the summation of all the involved sequences' autocorrelations.

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Besides the correlation properties, spectral compatibility is also increasingly considered in waveform design, due to the ever growing competition for spectrum [33–36]. Many methods [5,7,13,37–53] have been proposed to regulate the spectral shape, set nulls/notches, generate spectrum contained probing waveforms, and etc. to enable coexistence of multiple RF users. As the modification of waveform on spectral content likewise affects the correlation properties, most of these studies give considerations to both aspects simultaneously. For example, transmit sequence and receive filter are optimized to achieve sparse transmit and range sidelobes suppression in [40]. Transmit power in stopbands and the ISL are simultaneously minimized in [5,41–43]. Sequence is designed to approximate prescribed spectral and autocorrelation template in [44]. Detection performance optimization in colored noise with range sidelobes mask is considered in [45,46]. In [47–53], similarity constraint is enforced to control the AF features and modulus variations of the sought sequence, with the objectives being to enhance the detection probability and improve spectral compatibility in signal-dependent or signal-independent interference scenario.

Motivated by the widespread desires for good sequences (sets) as well as the direct/indirect spectral considerations involved, we develop a unified framework to design sequences with various desired spectral properties. We achieve the task by minimizing the residual error between the designed PSD and the target one. This unified metric includes the objective functions of PSD fitting, spectral mask reduction, improving signal-to-interference-plus-noise ratio (SINR), minimizing ISL, CSS design, and OSS design as special cases. In minimizing the PSD residual error, we restrict the designed sequence to be unimodular to benefit power efficiency and thus maximize the achievable sensitivity. We derive a way to employ fast Fourier transformation (FFT) in calculating the gradient of the PSD residual error with respect to (w.r.t.) sequence's phase, such that the PSD residual error can be efficiently minimized using general gradient method. The contributions of this paper can be summarized as follows: 1) We solve the sequence designs in a unified manner instead of dealing with each one separately. This unified framework provides a global understanding of the commonality of the considered design problems; 2) The FFT-based gradient calculation we derive makes the optimizing very computationally efficient and thus facilitates long sequence design and fast waveform adaption; 3) The proposed tool is very flexible and it can tackle the designs with composite objectives which are often encountered in practice and more difficult to solve using existing methods.

The reminder of this paper is organized as follows. In Section 2, we define the unified metric, i.e. the PSD residual error. We also discuss several sequence design scenarios and formulate their objective functions into the unified metric defined. In Section 3, we consider the minimization of the unified metric. We describe the conjugate gradient (CG) method employed, derive a way to employ FFT in calculating the gradient w.r.t. the sequence's phase, and illustrate how to tackle the designs with composite objectives. In Section 4, comparisons with state-of-the-art algorithms are made to validate the proposed method. Conclusion is drawn in Section 5.

1.1. Notation

Boldface lower case letters denote column vectors, boldface upper case letters denote matrices, and italics denote scalars. The superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote complex conjugate, transpose, and Hermitian transpose respectively. $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary part of the argument. $[\cdot]_{m,n}$ denote the (m, n)th entry of a matrix. $\mathbf{0}$ denotes the all-zero vector. The symbol $\odot$ stands for the Hadamard (element-wise) product of matrices. $\exp(\cdot)$ should be understood in a componentwise sense if the argument is a vector or matrix. $\mathbb{R}$ and $\mathbb{C}$ denote the real field and the complex field, respectively. $||\cdot||$ denotes the Frobenius norm in $\mathbb{R}^N$. The letter j represents the imaginary unit (i.e. $\mathrm{j}=\sqrt{-1}$), while the italic letter i, j, k, l, m, n, and t serve as index in this paper.

2. Applications and formulations

In this section, we will consider the sequence designs with different goals. Particularly, we will define a unified metric that includes the objective functions of the considered design problems as special cases, so that we only need to focus on the minimization of this unified metric in the subsequent optimization.

2.1. Unified metric

Let us consider a complex unimodular sequence of length N, i.e.

$$\boldsymbol{s} = [s(1), \cdots, s(N)]^{\mathrm{T}} = \frac{1}{\sqrt{N}} \exp(\mathrm{j}\boldsymbol{\varphi}) \tag{1}$$

where $\boldsymbol{\varphi}=[\varphi(1),\cdots, \varphi(N)]^{\mathrm{T}}$ is the phase vector. For the sequence $\boldsymbol{s}$, its K-point ($K \geq N$) PSD denoted by $\boldsymbol{p}$ is given by

$$\boldsymbol{p} = [p(1), \cdots, p(k)]^{\mathrm{T}} = \boldsymbol{f} \odot \boldsymbol{f}^* \tag{2}$$

$$\boldsymbol{f} = [f(1), \cdots, f(K)]^{\mathrm{T}} = \boldsymbol{F}\tilde{\boldsymbol{s}} \tag{3}$$

$$\tilde{\boldsymbol{s}} = \left[\boldsymbol{s}^{\mathrm{T}}\ \mathbf{0}_{1\times(K-N)}\right]^{\mathrm{T}} \tag{4}$$

where $\tilde{\boldsymbol{s}}$ is the zero-padded vector of $\boldsymbol{s}, \boldsymbol{f}$ is the frequency spectrum of $\boldsymbol{s}$, and $\boldsymbol{F}\in\mathbb{C}^{K\times K}$ is the $K\times K$ discrete Fourier transform (DFT) matrix with entries

$$\boldsymbol{F}_{k,n} = e^{-\mathrm{j}\frac{2\pi}{K}(n-1)(k-1)},\ 0 \leq n, k < K \tag{5}$$

Then the unified metric is defined as the following PSD residual error

$$E = \sum_{k=1}^{K} w_k [p(k) - d_k]^q \tag{6}$$

where $\boldsymbol{d}=[d_1,\cdots, d_K]^{\mathrm{T}}$ is the target PSD, $\boldsymbol{w}=[w_1,\cdots,w_K]^{\mathrm{T}}$ is the weighting vector, and q is the exponent.

2.2. PSD fitting

The most straightforward application of the unified metric (6) is to approximate a sequence's PSD to a prescribed target. And by properly selecting the exponent q, we can achieve the optimal fitting in different sense. For example, metric (6) effectively becomes the squared error when $q=2$. If we set the exponent q to be a very large **even** number, then minimizing (6) will be tantamount to minimizing the maximum PSD discrepancy. Moreover, the weighting vector $\boldsymbol{w}$ enables the designer to express a measure of the relative approximation accuracy in each FFT bin.

2.3. Lowering spectral mask

With the growing competition for spectrum in recent years [33–36], an important topic of spectrum management is to regulate the emission to make better use of this finite resource. The emissions are often required to sit within a prescribed spectrum mask with a very low upper bound in overlapped bands and steep roll-off rate in out-of-band (OOB) [33]. To achieve such goal, we can formulate the sequence design as the following minimax optimization, i.e.

$$\boldsymbol{\varphi} = \arg\min \max_{k\in\Omega} \{10\lg[p(k)] - H(k)\} \tag{7}$$