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Momentum fractional LMS for power signal parameter estimation

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ABSTRACT

Fractional adaptive algorithms have given rise to new dimensions in parameter estimation of control and signal processing systems. In this paper, we present novel fractional calculus based LMS algorithm with fast convergence properties and potential ability to avoid being trapped into local minima. We test our proposed algorithm for parameter estimation of power signals and compare it with other state-of-the-art fractional and standard LMS algorithms under different noisy conditions. Our proposed algorithm outperforms other LMS algorithms in terms of convergence rate and accuracy.

1. Introduction

Integer order adaptive signal processing algorithms have their benefits for many signal processing, physical processes and control applications. One well-known algorithm based upon gradient descent is Least Mean Square (LMS) algorithm [1]. Many variations of the standard LMS have been proposed in the literature to improve its convergence properties and estimation accuracy [2]. All these algorithms are based on integer order gradients which find the trajectory of the solution to the optimum value in the negative direction of the gradient.

Recently, a graceful number of research activities have emerged in applying fractional order calculus for the design of adaptive algorithms. The fractional order adaptive algorithms have shown improved performance in various engineering applications compared to integer order LMS based algorithms [3,4]. In this paper, we design a new fractional LMS algorithm with improved convergence properties as compared to standard LMS and state-of-the-art fractional LMS algorithms.

1.1. Related work

Fractional order calculus has equally evolved in parallel with integer order calculus in the field of mathematics. Its application in the field of sciences and engineering was initiated in [5]. Since then, it has been applied in a variety of domains where integer order adaptive algorithms were applied ranging from signal processing [6,7], biomedical problems [8], control [9,10], to physical processes [11,12]. The newly evolved fractional adaptive algorithms borrow their ideas from LMS algorithm and its variants by introducing different ways for step-size calculation and weights updating mechanisms. For example, fractional least mean square (FLMS) identification algorithm was developed by exploiting the theories of fractional calculus for weights update in standard LMS [3].

The FLMS update equation includes integer order gradient as well as the fractional order gradient. The trade-off between these two gradients is suggested in [13] that adds a proportion of each gradient according to the value of a forgetting factor. This results in better convergence as compared to the original FLMS in [3]. The convergence properties of FLMS is further improved by introducing a sliding window which also includes previous values of the input in addition to the current input values [14]. To reduce the computational complexity, works in [15] include only fractional part of the gradient in the weight update equation. By omitting the integer order gradient and retaining only the fractional part, the overall convergence is not affected but the computational complexity caused by the integer order gradient is reduced. The fractional order used in the algorithms so far lies in the range $\in (0,1)$ and as fractional order approaches to 1, convergence rate increases. However, higher fractional order also increases the steady state error. This behavior of rapidity and accuracy was further studied in [16] for fractional order $\in (1,1.5)$. The authors found that the same behavior of rapidity and accuracy is observed as in the original FLMS [3]. Modified LMS [17] was extended to fractional version in [18].

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The above discussion shows that different variants of LMS have been extended to fractional order and their properties were studied. In this study, we extend momentum LMS (mLMS) [19] to fractional order and empirically study its convergence properties and estimation accuracy for sinusoidal signal modeling. The mLMS updates the weights by incorporating proportion of the previously calculated gradients in the current update step. This increases the convergence rate of the mLMS as compared to the standard LMS. By incorporating these previously calculated gradients in the current weights update step of the standard FLMS algorithm [3], we also intend to improve the convergence properties of the FLMS and name it as momentum FLMS (mFLMS).

Many parameter estimation techniques exist in the literature for different applications [20–23]. Recently some new methods have been introduced in [24–28]. To demonstrate promising properties of the proposed method, we consider the application of signal modeling and parameter estimation of sinusiodal signals which are important for reliability assessment and quality monitoring of power systems. Frequency, as one of the parameters, is important to be estimated for harmonic measurement and compensation [29] and in phase lock loops (PLL) for grid signal synchronization with system output [30]. The amplitude estimate is used in fault detection algorithms [31] and in under/over voltage protection algorithms [32]. The phase estimate is used in different scenarios such as PLL algorithms [33] and in the generation of control signals in a controller [34]. Recently, a novel stochastic gradient algorithm has been proposed for estimating the parameters of the sine combination signal modeling, and further a multi-innovation stochastic gradient parameter estimation method is presented by expanding the scalar innovation into the innovation vector for improving the estimation accuracy [35]. Here we apply our proposed algorithm on parameter estimation problem of power signals and compare it with LMS, mLMS and FLMS.

1.2. Our contribution

Inspired by different variations in LMS to improve its convergence and parameter estimation properties [36], we also incorporate an adaptation term in standard fractional LMS (FLMS) [3] and study its convergence properties and estimation accuracy. We design momemtum fractional LMS (mFLMS) in which a momentum term is incorporated with standard FLMS that increases the speed of the convergence and has the ability to avoid trapping in local minima. This work is different from [36] where momentum term is used with simple (non-fractional) LMS (mLMS). To show the performance of the proposed algorithm, the mFLMS is applied to estimate magnitude and phase of a sinusoidal signal [35] which is a combination of different sinusoidal harmonics having different amplitudes and phases. We compare its performance with fractional LMS (FLMS), momemtum LMS (mLMS) and LMS algorithms with varying learning rate parameters and under different noise conditions. This is also different from the works in [14,15] where no momentum term is used.

1.3. Paper outline

The paper is organized as follows: Section 2 gives brief description about FLMS. Section 3 describes the design of mFLMS and its derivation for power signal. Section 4 gives experimental details followed by results and discussion. Finally, the paper is concluded in Section 6.

2. Fractional order least mean squares (FLMS)

Application of fractional calculus to standard LMS algorithm have given rise to FLMS algorithm [3] where apart from taking simple integer order derivative, the fractional order derivative is also used to calculate fractional order gradients for the minimization of objective function. Let $y(n)$ be the estimated signal, $d(n)$ be the desired signal and $e(n)$ be the error signal, then the objective function for the minimization of the error is:

$$J(n) = E[e(n)^2] = E[d(n) - y(n)]^2 \tag{1}$$

where $E[\cdot]$ is the expectation. The estimated output $y(n)$ is written as:

$$y(n) = \hat{\mathbf{w}}^T(n)\,\mathbf{u}(n) \tag{2}$$

where $\hat{\mathbf{w}}$ is the estimated weight vector and $\mathbf{u}$ is the input vector. To find the weights, we need to minimize objective function (1) with respect to $\hat{\mathbf{w}}$, given as:

$$\frac{\partial J(n)}{\partial \hat{\mathbf{w}}} = 2e(n)\frac{\partial e(n)}{\partial \hat{\mathbf{w}}} \tag{3}$$

Substituting $e(n)$ in above equation:

$$\frac{\partial J(n)}{\partial \hat{\mathbf{w}}} = 2e(n)\frac{\partial}{\partial \hat{\mathbf{w}}}\left(d(n) - \hat{\mathbf{w}}^T(n)\mathbf{u}(n)\right) \tag{4}$$

After simplifying (4):

$$\frac{\partial J(n)}{\partial \hat{\mathbf{w}}} = -2e(n)\mathbf{u}(n) \tag{5}$$

From (5), standard LMS update equation [1] is given by

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) - \frac{\mu_1}{2}\left(\frac{\partial J(n)}{\partial \hat{\mathbf{w}}}\right) \tag{6}$$

where μ_1 represents the step size parameter for standard LMS.

In Eq. (6), the first order gradient is used to update LMS weights. In case of the fractional LMS, in addition to first order gradient, fractional order gradient is also used. The recursive weight update relation for the fractional LMS algorithm is written as:

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) - \frac{\mu_1}{2}\left(\frac{\partial J(n)}{\partial \hat{\mathbf{w}}}\right) + \mu_f\left(\frac{\partial^f}{\partial \hat{\mathbf{w}}^f}J(n)\right) \tag{7}$$

where μ_f is the step size for the fraction order derivative ∂^f.

Following the Caputo and Riemann-Liouville definition [37], the fractional derivative of a function $g(t) = t^n$ is defined as:

$$D^f g(t) = \frac{\Gamma(n+1)}{\Gamma(n-f+1)}t^{n-f} \tag{8}$$

where D^f is fractional derivative operator of order f and Γ is a gamma function, defined as:

$$\Gamma(n) = (n-1)! \tag{9}$$

By using the above definitions for fractional order derivatives, the fractional order derivative in (7) becomes:

$$\frac{\partial^f}{\partial \hat{\mathbf{w}}^f}J(n) = -2(e(n)\mathbf{u}(n))\left(\frac{\partial^f}{\partial \hat{\mathbf{w}}^f}\hat{\mathbf{w}}(n)\right) \tag{10}$$

By using (8), (10) becomes:

$$\frac{\partial^f}{\partial \hat{\mathbf{w}}^f}J(n) = -2(e(n)\mathbf{u}(n))\left(\frac{\Gamma(2)}{\Gamma(2-f)}\hat{\mathbf{w}}^{1-f}(n)\right) \tag{11}$$

As $\Gamma(2) = 1$, substituting (11) in (7), we have:

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \mu_1 e(n)\mathbf{u}(n) + \frac{\mu_f}{\Gamma(2-f)}e(n)\mathbf{u}(n) \odot |\hat{\mathbf{w}}|^{1-f}(n) \tag{12}$$

where the symbol $\odot$ denotes an element by element multiplication of vectors and the absolute value of vector $\hat{\mathbf{w}}$ is used to avoid complex values.

Eq. (12) is the weight update equation of the standard FLMS algorithm [3].