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Time-frequency decomposition of multivariate multicomponent signals

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ABSTRACT

A solution of the notoriously difficult problem of characterization and decomposition of multicomponent multivariate signals which partially overlap in the joint time-frequency domain is presented. This is achieved based on the eigenvectors of the signal autocorrelation matrix. The analysis shows that the multivariate signal components can be obtained as linear combinations of the eigenvectors that minimize the concentration measure in the time-frequency domain. A gradient-based iterative algorithm is used in the minimization process and for rigor, a particular emphasis is given to dealing with local minima associated with the gradient descent approach. Simulation results over illustrative case studies validate the proposed algorithm in the decomposition of multicomponent multivariate signals which overlap in the time-frequency domain.

1. Introduction

Signals with time-varying spectral content are not readily characterized by the conventional Fourier analysis, and are commonly studied within the time-frequency (TF) analysis [1–8]. Research in this field has resulted in numerous representations and algorithms which have been almost invariably introduced for the processing of univariate signals, with most frequent characterization through amplitude and frequency-modulated oscillations [6,9].

Recently, the progress in sensing technology for multidimensional signals has been followed by a growing interest in time-frequency analysis of such multichannel (multivariate and/or multidimensional) data. Namely, developments in sensor technology have made accessible multivariate data. Indeed, the newly introduced concept of modulated bivariate and trivariate data oscillations (3D inertial body sensor, 3D anemometers [9]) and the generalization of this concept to an arbitrary number of channels have opened the way to exploit multichannel signal interdependence in the joint time-frequency analysis [10–12].

The concept of multivariate modulated oscillations has been proposed in [10], under the restricting assumption that one common oscillation fits best all individual channel oscillations. In other words, a joint instantaneous frequency (IF) aims to characterize multichannel data by capturing the combined frequency of all individual channels. It is defined as a weighted average of the IFs in all individual channels. The deviation of multivariate oscillations in each channel from the joint IF is characterized by the joint instantaneous bandwidth. With the aim to estimate the joint IF of multichannel signals, the synchrosqueezed transform, a highly concentrated time-frequency representation (TFR) belonging to the class of reassigned TF techniques, has been recently extended to the multivariate model [9]. Following the same aim of extracting the local oscillatory dynamics of a multivariate signal, the wavelet ridge algorithm has also been introduced within the multivariate framework [10]. Another very popular concept, empirical mode decomposition (EMD), has been studied for multivariate data, [18–22]. However, successful EMD-based multicomponent signal decomposition is possible only for signals which exhibit nonoverlapping components in the TF plane.

By virtue of high concentration and many other desirable properties, the Wigner distribution is commonly exploited in numerous IF estimators developed within the TF signal analysis [6–8]. However, in the case of multicomponent signals, undesirable oscillatory interferences known as cross-terms appear, sometimes masking the presence of desirable auto-terms. To this end, other representations have been developed, commonly aiming to preserve Wigner distribution concentration, while suppressing the cross-terms. One such algorithm is the S-method [6] which was also used as a basis for the multi-component signal decomposition algorithm, proposed in [1]. This particular type of decomposition makes it possible to analyze and characterize signal components independently, allowing the IF estimation for each separate component [1–4].

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In this paper, multivariate Wigner distribution is studied as the basis of multicomponent multichannel signal decomposition. Namely, the strong interdependence of modulations of individual components within all the available data channels is exploited in the joint TF analysis, leading to a reduction of undesirable oscillations present in cross-terms. The inverse multivariate Wigner distribution matrix is decomposed into eigenvectors which contain signal components in the form of their linear combination. Further, a steepest-descent algorithm that enables a fast search for a linear combination of eigenvectors that produces the best possible components concentration is applied. Using the advantages of multichannel interdependence, the proposed TF-based decomposition is shown to be successful in the case of multivariate signals which overlap in the TF plane, while preserving the integrity of each extracted signal component.

Notice that the conventional time-frequency decomposition techniques cannot separate crossing components of arbitrary forms, which may appear in various signal processing applications. One such scenario is in radar signal processing, where reflecting points may assume the same velocity along the line-of-sight. These components will cross in the time-frequency (time-Doppler) representation. The same effect appears when the target signature crosses with the clutter or stationary body reflecting component in the time-frequency representation of radar signal return. The proposed method assumes that multiple phase independent received signals are available. They can be obtained using polarization or multiple antenna systems [23]. Signals with low frequency variations, when the amplitude changes are of the same order as the phase changes, can also be treated as signals with crossing components. Such are the ECG signals, for example. Multivariate forms of these signals are obtained using multiple sensors at different locations. The presented approach can be applied to the decomposition of this class of signals as well.

The paper is organized as follows. Basic theory regarding multivariate TF signal analysis is presented in Section 2. In Section 3, the Wigner distribution of multivariate multicomponent signals is analyzed. In Section 4, we present the basic theory leading to the decomposition of multivariate multi-component signals, whereas the decomposition algorithm is presented in Section 5. The theory is verified through several numerical examples in Section 6.

2. Multivariate time-frequency analysis

Consider a multivariate signal

$$\mathbf{x}(t)=\begin{bmatrix} a_1(t)e^{j\phi_1(t)} \\ a_2(t)e^{j\phi_2(t)} \\ \vdots \\ a_N(t)e^{j\phi_N(t)} \end{bmatrix} \tag{1}$$

obtained by measuring a complex-valued signal $x(t)$ with N sensors, where by each sensor the amplitude and phase of the original signal are modified to give $a_i(t)\exp(j\phi_i(t))=\alpha_i x(t)\exp(j\varphi_i)$. If the measured signal is real-valued, its analytic extension

$$x(t)=x_R(t)+j\mathrm{H}\{x_R(t)\}$$

is commonly used, with $x_R(t)$ being real-valued measured signal and $\mathrm{H}\{x_R(t)\}$ its Hilbert transform. Analytic signal contains only nonnegative frequencies and the real-valued counterpart can be reconstructed. This form of signal is especially important in the instantaneous frequency interpretation within the time-frequency moments framework.

Since all time-frequency representations may be considered as smoothed versions of the Wigner distribution, this distribution will be the starting point for a review of time-frequency based multivariate signal analysis. The Wigner distribution of a multivariate signal $\mathbf{x}(t)$ is defined as

$$WD(\omega,t)=\int_{-\infty}^{\infty}\mathbf{x}^H(t-\frac{\tau}{2})\mathbf{x}(t+\frac{\tau}{2})e^{-j\omega\tau}d\tau, \tag{2}$$

where $\mathbf{x}^H(t)$ is a Hermitian transpose of the vector $\mathbf{x}(t)$.

The inverse Wigner distribution is then given by

$$\mathbf{x}^H(t-\frac{\tau}{2})\mathbf{x}(t+\frac{\tau}{2})=\frac{1}{2\pi}\int_{-\infty}^{\infty}WD(\omega,t)e^{j\omega\tau}d\omega. \tag{3}$$

The center of mass in the frequency axis of the Wigner distribution of a multivariate signal $\mathbf{x}(t)$, defined by (1), is given by

$$\langle\omega(t)\rangle=\frac{\int_{-\infty}^{\infty}\omega WD(\omega,t)d\omega}{\int_{-\infty}^{\infty}WD(\omega,t)d\omega}$$

or, more explicitly

$$\langle\omega(t)\rangle=\frac{\frac{d}{jd\tau}\left[\mathbf{x}^H(t-\frac{\tau}{2})\mathbf{x}(t+\frac{\tau}{2})\right]_{|\tau=0}}{\mathbf{x}^H(t-\frac{\tau}{2})\mathbf{x}(t+\frac{\tau}{2})_{|\tau=0}}=\frac{1}{2j}\frac{[\mathbf{x}^H(t)\mathbf{x}'(t)-\mathbf{x}'^H(t)\mathbf{x}(t)]}{\mathbf{x}^H(t)\mathbf{x}(t)},$$

where $\mathbf{x}'(t)=d\mathbf{x}(t)/dt$ denotes derivative in time.

The expression for instantaneous frequency of a multivariate signal follows straightforwardly from the previous relation in the form:

$$\langle\omega(t)\rangle=\frac{\sum_{n=1}^{N}\phi_n'(t)a_n^2(t)}{\sum_{n=1}^{N}a_n^2(t)}. \tag{4}$$

If a multivariate signal is obtained by sensing a monocomponent signal $x(t)$ as $a_i(t)\exp(j\phi_i(t))=\alpha_i x(t)\exp(j\varphi_i)$ with $x(t)=A(t)\exp(j\psi(t))$ and $|dA(t)/dt|\ll|d\psi(t)/dt|$, then $\langle\omega(t)\rangle=d\psi(t)/dt$, since $d\phi_i(t)/dt=d\psi(t)/dt$. The condition for amplitude and phase variations of real-valued monocomponent signals $a_i(t)\cos(\phi_i(t))$ can be defined by Bedrosian's product theorem [13]. It states that the complex analytic signal $a_i(t)\exp(j\phi_i(t))=a_i(t)\cos(\phi_i(t))+j\mathrm{H}\{a_i(t)\cos(\phi_i(t))\}$ is a valid representation of the real amplitude-phase signal $a_i(t)\cos(\phi_i(t))$ if the spectrum of $a_i(t)$ is nonzero only within the frequency range $|\omega|<B$ and the spectrum of $\cos(\phi_i(t))$ occupies nonoverlapping higher frequency range. A signal is monocomponent if the spectrum of $a_i(t)$ is of lowpass type.

This analysis can be generalized to other time-frequency and time-scale signal representations.

A deviation of the signal spectral content from the instantaneous frequency is described by the local second order moments (instantaneous bandwidths). The expression for the instantaneous bandwidth is obtained from

$$\sigma_\omega^2(t)=\frac{1}{2\pi\mathbf{x}^H(t)\mathbf{x}(t)}\int_{-\infty}^{\infty}\omega^2WD(t,\omega)d\omega-\langle\omega(t)\rangle^2=\frac{-\frac{d^2}{d\tau^2}\left[\mathbf{x}^H\left(t-\frac{\tau}{2}\right)\mathbf{x}\left(t+\frac{\tau}{2}\right)\right]\Big|_{\tau=0}}{\mathbf{x}^H(t)\mathbf{x}(t)}-\langle\omega(t)\rangle^2.$$

For the signal in (1) it has the following form:

$$\sigma_\omega^2(t)=\frac{\sum_{n=1}^{N}(a_n'(t))^2-\sum_{n=1}^{N}a_n(t)a_n''(t)}{2\sum_{n=1}^{N}a_n^2(t)}.$$

In general, for the case of multicomponent signals, the components are localized over more than one instantaneous frequency.

3. Multicomponent signals

Consider a multicomponent signal

$$x(t)=\sum_{p=1}^{P}x_p(t)$$