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Stochastic topology design optimization for continuous elastic materials

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Highlights

- Propose a new mathematical framework for stochastic topology optimization.
- Study of the stability of structures submitted to several loads.
- Introduce a Variance-Expected compliance model for topology optimization.
- Deterministic formulation of the proposed model.
- Perform a numerical validation of the model considering 2D and 3D benchmark cases.

Abstract

In this paper, we develop a stochastic model for topology optimization. We find robust structures that minimize the compliance for a given main load having a stochastic behavior. We propose a model that takes into account the expected value of the compliance and its variance. We show that, similarly to the case of truss structures, these values can be computed with an equivalent deterministic approach and the stochastic model can be transformed into a nonlinear programming problem, reducing the complexity of this kind of problems. First, we obtain an explicit expression (at the continuous level) of the expected compliance and its variance, then we consider a numerical discretization (by using a finite element method) of this expression and finally we use an optimization algorithm. This approach allows to solve design problems which include point, surface or volume loads with dependent or independent perturbations. We check the capacity of our formulation to generate structures that are robust to main loads and their perturbations by considering several 2D and 3D numerical examples. To this end, we analyze the behavior of our model by studying the impact on the optimized solutions of the expected-compliance and variance weight coefficients, the laws used to describe the random loads, the variance of the perturbations and the dependence/independence of the perturbations. Then, the results are compared with similar ones found in the literature for a different modeling approach.

MSC: 35Q74; 74B05; 74P05; 74S60

Keywords: Topology optimization; Structural optimization; Stochastic programming; Finite element method

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1. Introduction

Let us consider an open set $\Omega \subset \mathbb{R}^d$, where d is 2 for planar structures or 3 for three-dimensional bodies. The set Ω represents a body that we assume is made of an isotropic, homogeneous and linear elastic material. The boundary of Ω is denoted here by $\partial\Omega = \Gamma_u \cup \Gamma_g$ with $\Gamma_u \cap \Gamma_g = \emptyset$. In this setting, Γ_u corresponds to the part of the boundary of Ω where the displacements of the body are not allowed. We assume that external forces f and g are applied to Ω and Γ_g, respectively. A graphical representation of Ω is given in Fig. 1.

The displacements can be computed (see e.g. [1]) by solving the following system of partial differential equations:

$$\begin{cases} -\operatorname{div}(K\, e(u)) = f, & \text{in } \Omega, \\ u = 0, & \text{on } \Gamma_u, \\ (K\, e(u)) \cdot \hat{n} = g, & \text{on } \Gamma_g, \end{cases} \tag{1}$$

where $u\colon \Omega \to \mathbb{R}^d$ is the vector of displacements, $e(u) = \frac{1}{2}(\nabla u + \nabla u^t)$ denotes the linearized strain tensor, K is the fourth-order material elasticity tensor, $\operatorname{div}(\cdot)$ is the divergence of a tensor field and $\hat{n}$ is the outward unit normal vector on the boundary of the domain. We suppose that $K \in M$, which is a set of admissible stiffness tensors, related to the admissible materials we might use. Typically (see [2,3]), the set of admissible tensor M is a subset of

$$\hat{M} = \left\{ \eta(x) K^0 \;\middle|\; \int_\Omega \eta(x)\, \mathrm{d}x \le V_{\max} \text{ and } \eta\colon \Omega \to [\eta_{\min}, \eta_{\max}] \right\}, \tag{2}$$

where $V_{\max}$ is the maximum amount of material that is allocated; $\eta(x)$ is the density of material at point $x \in \Omega$; $\eta_{\min} > 0$, $\eta_{\max}$ the maximum density we may use; and K^0 the fourth-order tensor of a linear elastic isotropic reference material. Additionally, other constraints should be considered in order to obtain physically realizable structures. For example, we would like to avoid structures with intermediate density zones (i.e., $\eta(x) \in\ ]\eta_{\min}, \eta_{\max}[$), and microstructures with periodic variation density. For practical reasons, these constraints are not detailed in the continuous definition of M but they will be taken into account in its numerical implementation (see Section 3).

In the following, we assume that $f \in L^2(\Omega)^d$ and $g \in L^2(\Gamma_g)^d$ although less regular external forces can also be considered (see Remark 1)

Let us define $H = \{u \in [H^1(\Omega)]^d : u|_{\Gamma_u} = 0\}$, where the space $H^1(\Omega)$ is the well-known Sobolev space of functions that are in $L^2(\Omega)$ with the first derivatives (in the sense of distributions) in $L^2(\Omega)$. For a given material and its corresponding stiffness tensor $K \in M$, following [2,4] we define the bilinear functional $A_K\colon H \times H \to \mathbb{R}$ by

$$A_K(u, v) := \int_\Omega e(u) : K e(v)\, \mathrm{d}x, \tag{3}$$

where $e(u) : K e(v)$ denotes the tensor product given by

$$e(u) : K e(v) := \sum_{i,j,k,l=1}^{d} K_{ijkl}\, e_{ij}(u) e_{kl}(v).$$

We recall that a weak solution of system (1) is a vector $u \in H$ satisfying

$$A_K(u, v) = \int_\Omega f \cdot v\, \mathrm{d}x + \int_{\Gamma_g} g \cdot v\, \mathrm{d}x \quad \forall v \in H. \tag{4}$$

We note that, under suitable conditions on the data (according to Korn's inequality and the Lax–Milgram Lemma [5,6]), Problem (1) has a unique weak solution (see [7] for more details).

Remark 1. In this work it is possible to include functions less regular than $f \in L^2(\Omega)$ or $g \in L^2(\Gamma_g)$ with a suitable alternative weak formulation instead of (4). An example of force $f \notin L^2(\Omega)$ that is typically considered is a point wise force $f = \bar{f}\delta(x - a)$, where $\bar{f} \in \mathbb{R}^d$ and $a \in \bar{\Omega}$. In this case, $\int_\Omega f \cdot v\, \mathrm{d}x = \bar{f} v(a)$. Nevertheless, when computing an approximated solution by using the Finite Element Method (see Section 3) f and g are usually approximated by functions in $L^2(\Omega)^d$ and $L^2(\Gamma_g)^d$, respectively, and (4) can be used again, becoming a linear system in finite dimension.