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ϕ-linear canonical analytic signals

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ABSTRACT

In this paper, a ϕ-linear canonical Hilbert transform (ϕ-LCHT) is proposed by a linear combination of a given signal and its parameter Hilbert transform (PHT). Herein, a PHT is a generalized HT based on the linear canonical transform (LCT). Some essential properties of a ϕ-LCHT are also derived. According to the derivation of a ϕ-LCHT, four different versions of ϕ-linear canonical analytic signals (ϕ-LCASs) are defined and analyzed in detail. Furthermore, ϕ-LCASs are discussed and compared by some valuable properties such as the LCT spectrum and ϕ-LCHT eigenfunction property. In terms of application, a multi-key single-sideband (SSB) modulation scheme is investigated using one of the ϕ-LCASs proposed in this paper, and its performance with respect to noise and sensitivity to security key perturbations is analyzed.

1. Introduction

The Hilbert transform (HT) plays an important role in signal processing. It was first introduced into the communication field by Gabor in 1946 [1]. He constructed a complex signal called an analytic signal (AS) by adding a given real signal in quadrature with its HT. One of the most important properties of an AS is that it contains no negative Fourier frequency, which is commonly referred to as the one-sided spectrum. This property is significant in single-sideband (SSB) modulation applications where an AS can reduce the signals transmission bandwidth by half. In addition, an AS is also an eigenfunction of the HT operator. The HT and AS have many interesting and important applications including image edge enhancement [2], instantaneous frequency estimation [3,4], spectral analysis [5], sampling rate conversation [6], and time delay estimation [7,8].

The classical AS is closely related to the Fourier transform (FT) and HT. Since FT analysis is limited for non-stationary signals, more and more novel transformation methods have recently been proposed such as the wavelet transform (WT) [9], fractional Fourier transform (FrFT) [10], and linear canonical transform (LCT) [11,12]. With respect to the aforementioned applications of the AS and HT and the flexibility of these emerging approaches over the FT, a large number of attempts have been made to improve the performances of both the HT and AS by extending them into new transformed domains. In 1998, Zayed combined the HT with the FrFT and proposed the fractional Hilbert transform (FrHT). A generalization of the AS was also provided by suppressing the negative frequency of the FrFT signal [13]. However, this extension of the HT and AS lacks the semi-group property of the FrFT. Fu and Li extended the HT into the LCT domain and proposed the parameter Hilbert transform (PHT). In their work, they presented a generalized analytic signal (GAS) by suppressing the negative frequency components in the LCT domain [14]. They also derived a generalized Bedrosian theorem related to the LCT [14,15]. Note however, that the definition of the GAS by Fu et al. has some drawbacks. In particular, for the classical AS, it is sufficient to carry all of the signal information but only with half-bandwidth, mainly due to the fundamental conjugate symmetry property of the FT for a real signal. On the contrary, the LCT for a given real signal lacks this important property. Thus, only by suppressing the negative portion of LCT-transformed signals, the original signal can hardly recovered from the corresponding GAS. To address this drawback, Pei proposed another AS based on the LCT called the canonical analytic signal (CAS) [16]. Moreover, two-dimensional HTs associated with the LCT were proposed by Xu et al. in a different way. They also investigated generalized uncertainty principles related to the HT and LCT [17,18]. Discrete implementations for the HT and its generalized forms are provided in the literature [19–21].

Another way to improve the HT is by adding a rotation parameter in terms of the filter function [22], which is different from transform techniques. It possesses the semi-group property, while fractional analytic signals (FASs) associated with the rotation parameter [23] cannot always contain a one-sided spectrum. On the other hand, the generalized-phase analytic signal (GPAS) was proposed by Venkitaraman et al. based on the generalized-phase Hilbert transform (GPHT) [24]. The phase of the GPHT in

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the Fourier domain is not constant, but varies over the Fourier frequency. Both the HT and its extensions are valuable components in AS (or generalized AS) construction and play crucial roles in many practical applications such as SSB modulation and image enhancement. In consideration of the drawbacks of the existing methods, it is necessary to develop a better definition. The main goal of this paper is to propose a new definition of the HT, combining the rotation parameter with the LCT; specifically, it is a rotation-weighted original signal and its generalized HT based on the LCT called as ϕ-linear canonical Hilbert transform (ϕ-LCHT). It can involve most extensions of the HT as its special cases and satisfy specific properties. Some important properties of the ϕ-LCHT are discussed. Based on the rotation parameter and LCT, four different extensions of the AS are proposed and analyzed, which we refer to as ϕ-linear canonical analytic signals (ϕ-LCASs). In addition, one of ϕ-LCASs is successfully applied to a SSB modulation application and the performance of experimental results is discussed in detail.

The remainder of this paper is organized as follows. Section 2 provides some basic knowledge of related work on the HT and AS. In Section 3, the definition and properties of the ϕ-LCHT are proposed. In addition, derivations of the ϕ-LCHT are analyzed from two different views as well. Four ϕ-LCASs are presented in Section 4. The differences between these new analytic signals are compared in terms of the LCT spectrum characteristic and eigenfunction property of the ϕ-LCHT. In order to better understand ϕ-LCASs, Section 5 summarizes their merits and conducts further investigations using the Wigner–Ville distribution (WVD) in the time-frequency plane. Finally, applications and conclusions are stated in Sections 6 and 7, respectively.

2. Preliminaries

2.1. Hilbert transform and analytic signal

The HT of a real-valued signal $s(t)$ is defined by [1]

$$\tilde{s}(t) = \mathscr{H}[s](t) = p.v.\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{s(\tau)}{t-\tau}d\tau = (s*h)(t), \tag{1}$$

where $\mathscr{H}$ is the HT operator, p.v. denotes the Cauchy principal value integral, $h(t) = 1/\pi t$, and $*$ is the convolution operator. The corresponding AS of $s(t)$ is

$$z(t) = s(t) + j\tilde{s}(t) = s(t) + j\mathscr{H}[s](t). \tag{2}$$

In the Fourier domain, the AS is represented as

$$Z(w) = (1 + sgn(w))S(w) = 2U(w)S(w), \tag{3}$$

where w is the Fourier frequency, $sgn(\cdot)$ denotes the signum signal, $U(\cdot)$ is the unit-step signal, and $Z(w)$, $S(w)$ represents the FT of $z(t)$, $s(t)$, respectively. Eq. (3) shows that the AS has only a one-sided spectrum in the Fourier domain, which plays an important role in wide practical applications such as SSB modulation and filter design.

2.2. Fractional Hilbert transform and analytic signals

Because the HT is orthogonal to the original signal, it lacks some effectiveness in practical applications. For example, it cannot be used to selectively enhance edges during image processing [2]. Thus, it has been improved by researchers by adding a rotation parameter ϕ. The fractional Hilbert transform (FHT) is defined as a linear combination of $s(t)$ and $\tilde{s}(t)$ [23]:

$$\tilde{s}_\phi(t) = \mathscr{H}_\phi[s](t) = \cos\phi\, s(t) + \sin\phi\, \tilde{s}(t), \tag{4}$$

where $\mathscr{H}_\phi$ denotes the FHT operator. Clearly, when $\phi = \pi/2$, $\tilde{s}_\phi(t)$ yields $\tilde{s}(t)$.

According to parameter ϕ, three different fractional analytic signals (FASs) were proposed by Cusmariu [23]:

$$\begin{aligned} z1_\phi(t) &= \mathscr{H}_\phi[z](t) \\ z2_\phi(t) &= s(t) + j\mathscr{H}_\phi[s](t) \\ z3_\phi(t) &= \cos\phi\, s(t) + j\sin\phi\, \mathscr{H}[s](t). \end{aligned} \tag{5}$$

Some properties of the aforementioned FASs in Fourier domain are discussed in detail such as instantaneous frequency, instantaneous amplitude, and instantaneous phase [23].

2.3. Parameter Hilbert transform and generalized analytic signal

Another method based on a transform technique to improve the effectiveness of the HT and AS is introduced in this subsection. Different from the FHT and FASs added to a free rotation parameter ϕ, the parameter Hilbert transform (PHT) and generalized analytic signal (GAS) were proposed by Fu and Li [14], which are closely related to a generalized Fourier transform, i.e., the LCT (see Section 2.4).

As the generalizations of HT and AS associated with the LCT, PHT and GAS are defined by [14]

$$\tilde{s}_A(t) = \mathscr{H}_A[s](t) = \frac{e^{-j\frac{a}{2b}t^2}}{\pi}\int_{-\infty}^{+\infty}\frac{s(\tau)e^{j\frac{a}{2b}\tau^2}}{t-\tau}d\tau \tag{6}$$

and

$$z_A(t) = s(t) + j\tilde{s}_A(t) = s(t) + j\mathscr{H}_A[s](t), \tag{7}$$

where $\mathscr{H}_A$ is the PHT operator. Eqs. (6) and (7) reduce to the classical form, HT and AS, respectively, under specific parameters since the LCT equals the FT with $A = (0, 1; -1, 0)$. Similarly, the PHT and GAS reduces to the FrHT and FrAS proposed by Zayed, respectively, with $A = (\cos\alpha, \sin\alpha; -\sin\alpha, \cos\alpha)$ [13].

The LCT of the PHT and GAS when $b > 0$ is

$$\mathscr{L}^A_{\tilde{s}_A}(u) = -jsgn(u)S^A(u) \tag{8}$$

and

$$\mathscr{L}^A_{z_A}(u) = (1 + sgn(u))S^A(u) = 2U(u)S^A(u), \tag{9}$$

respectively. From Eq. (9), the GAS is obtained by suppressing the negative frequency content of the LCT; that is, it contains a one-sided spectrum in the LCT domain.

2.4. The linear canonical transform

The LCT of a signal $s(t)$ with parameter matrix A is defined by [11,12]

$$S^A(u) = \mathscr{L}^A_s(u) = \begin{cases} \int_{-\infty}^{+\infty} s(t)K_A(u,t)dt, & b \neq 0 \\ \sqrt{d}e^{j(cd/2)u^2}s(du), & b = 0 \end{cases}, \tag{10}$$

where $K_A(u, t)$ denotes the LCT kernel and is given by

$$K_A(u,t) = \frac{1}{\sqrt{j2\pi b}}e^{j\left(\frac{du^2}{2b} - \frac{ut}{b} + \frac{at^2}{2b}\right)} \tag{11}$$

with parameter matrix $A = (a, b; c, d)$, where a, b, c, and d are real numbers satisfying $ad - bc = 1$, i.e., $det(A) = 1$.

The inverse linear canonical transform (ILCT) is another LCT whose parameter matrix equals the inverse matrix of the LCT with A. It can be rewritten as [11,12]

$$s(t) = \mathscr{L}^{A^{-1}}_{S^A}(t) = \begin{cases} \int_{-\infty}^{+\infty} S^A(u)K_{A^{-1}}(t,u)du, & b \neq 0 \\ \sqrt{a}e^{j(-ca/2)t^2}S^A(at), & b = 0 \end{cases}, \tag{12}$$

where $A^{-1} = (d, -b; -c, a)$ is the inverse matrix of A.

The LCT is particularly powerful in various applications such as solving differential equations, optical systems, quantum mechanics and image processing [25,26]. Many well-known signal processing operations, involving the FT, FrFT, Fresnel transform, Lorentz transform, and scaling operation are special cases of the LCT [27].