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Stress constrained topology optimization with free-form design domains

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Abstract

This paper aims at dealing with realistic and challenging design problems of stress constrained topology optimization with free-form design domains. First, the concept of level set function (LSF) based modelers is introduced to transform this kind of problems into the Boolean conjunction operation of a topology variation modeler (TVM) onto a free-form design domain modeler (FDDM). Such an operation is mathematically realized by means of the so-called R-functions in the form of implicit LSFs. Within this framework, topology optimization problems are classified into two general cases depending upon the existence of non-designable solid feature. Analytical sensitivity analysis formulas are further derived. Compared with the existing level set based method, the important sensitivity property of design domain preserving makes it possible to avoid automatically the boundary violation of the design domain caused by the zero level set movement and both the topology and boundary shape of the free-form design domain can be simultaneously optimized. Second, the implementation of the finite cell method (FCM) ensures the stress computing accuracy in the fixed mesh due to the use of high-order shape functions and adaptive integration scheme. The combination of the active-set strategy and the dynamic aggregation technique also reduces the number of local stress constraints greatly. Finally, representative examples are presented to illustrate the conveniences and effectiveness of the proposed method.

Keywords: Topology optimization; Stress constraints; Free-form design domain modeler; Topology variation modeler; Finite cell method

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1. Introduction

Topology optimization, with the aim of finding the optimal material distribution over a specific design domain, has been recognized as a promising approach to obtain innovative conceptual designs without any prior knowledge of the final structures [1–3]. Earlier studies were mostly devoted to the enhancement of global structural performances e.g., the mean compliance in the formulation of objective function and design constraints. In recent years, much attention has been paid to local stress constraints owing to the practical strength designs. The majority of researches

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concerned regular design domains rather than free-form ones and the classical Solid Isotropic Microstructure with Penalization (SIMP) method [4–8] was considered as the basic method.

As illustrated by Wang and Li [9], the SIMP based methods have to tackle the singularity phenomenon caused by the topology-dependent stress constraint function and especially the accuracy issue of local stress analysis near the boundaries. Comparatively, the LSF/XFEM optimization framework [9–12] has the advantages of describing the boundaries smoothly and guaranteeing the stress accuracy near the boundaries without gray regions and global mesh updating. Another important issue refers to the huge number of local stress constraints. To circumvent this difficulty, the active-set method and the aggregation method were applied independently. In detail, the former only takes into account potentially critical stress constraints during the optimization process [4,8,13], and the latter replaces the stress constraints with a single global constraint or a relatively small number of block aggregated constraints via aggregation functions such as p-norm measure function, Kreisselmeier–Steinhauser (K–S) function and their enhanced versions [5,6,14,15]. It should be indicated that although the combination of both methods could effectively control the local stress level, many internal parameters should be set in advance [7]. Otherwise, the stress convergence oscillates during the whole optimization process [9,16].

The stress constrained topology optimization confined to free-form design domains is a challenging problem that reflects the real needs of practical applications. Compared with the so-called ground structure of regular design domain, a real structure has, in fact, a design domain limited with boundaries of free forms that are more complicated than conventionally studied rectangle, L-shape and others. Meanwhile, as the standard SIMP method basically resorts to the fixed finite element mesh, it is hard to ensure the stress accuracy for design domains of changeable boundaries without mesh updating. The material perturbation technique [17,18] could be used to simplify the sensitivity analysis scheme.

In fact, the conventional level set based topology optimization resorted to the discrete form in terms of nodal values of a finite element model [19,20] and could not handle free-form design domains easily because corresponding Hamilton–Jacobi equations are solved by the finite difference scheme working only over a structured grid. Xing et al. [21] adopted the finite element based level-set method for irregular design domains, but it has nearly the same defect as the SIMP method when treating design domains with moving boundaries. Zhou and Wang [22] proposed the constructive solid geometry based level set description. Chen et al. [23] constructed B-splines and R-functions based level set description [24,25]. Both works were only limited to the compliance related topology optimization with free-form design domains and non-designable solid features involved in the design domain were not considered comprehensively. James and Martins [26,27] introduced the isoparametric concept into the stress related topology optimization for those design domains whose shape and configuration are easily constructed by mapping operations from intact rectangular regions.

Based on the above works, the main contributions of this paper are as follows. The FCM [28–31] is implemented to use a fixed grid for stress analysis. Thanks to the use of high-order B-spline shape functions and quadtree cell refinement, the computing accuracy of the FCM is much higher than that of the conventional FEM widely used in topology optimization. Its computing efficiency is also superior to that of the simplified XFEM version employed recently in level set based topology optimization [9,11]. The topology optimization process is transformed as an action of the topology variation modeler (TVM) onto the free-form design domain modeler (FDDM). In detail, the TVM is devised as parameterized LSFs, e.g., interpolations of RBFs and B-splines whose coefficients are considered as design variables for topology optimization. The FDDM can be constructed using the LSFs of relevant primitives constituting the free-form design domain, and it could be realized by means of R-functions, K–S function, Ricci function [32], etc. As a result, both modelers are combined through Boolean set operations introduced in Section 2.3 to confine the topology optimization to predefined free-form design domains. In this paper, R-functions and CS-RBFs are chosen as functionalities of the FDDM and TVM respectively to deal with topology optimization within free-form design domains. This is the main difference from our previous work [33] concerning the regular design domains in favor of the definition of CS-RBFs. The active-set method [13] is jointed with the dynamic aggregation method [34] to control the local stress level with merits of low computation cost, high precision and good convergence property. Two general design cases are systematically classified according to the needs of feature preserving in practical designs. The formulated sensitivity analysis indicates that the zero level set movement can be really limited within the free-form design domains.

This paper is organized as follows. Section 2 introduces the two LSF based modelers and their integration for the topology optimization with free-form design domains. Section 3 provides a brief introduction about the FCM dedi-