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Design perfect reconstruction cosine-modulated filter banks via quadratically constrained quadratic programming and least squares optimization

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ABSTRACT

In this paper, the design of perfect reconstruction (PR) cosine-modulated filter banks (CMFBs) is implemented via quadratically constrained quadratic programming (QCQP) and least squares (LS) optimization. To this end, a PR CMFB design problem is formulated as a nonconvex QCQP after re-arranging the coefficients of the prototype filter. Then a deep insight is offered into the algebraic relationship between the PR conditions and near-perfect reconstruction (NPR) ones for CMFB designs. Here we theoretically show that the NPR conditions are just the summations of the PR conditions. Firmly in the light of this relationship, a two-stage method is proposed for PR CMFB design. We firstly solve an NPR CMFB problem to obtain its optimal solution as a reference point, then model the PR CMFB design problem as a series of small-sized LS problems near the reference point. And we solve the LS problems in parallel with cheap iteration. Our analysis and numerical results show that the proposed method bears superior performance on effectiveness and efficiency, especially in the case of designing PR CMFBs with large number of channels.

1. Introduction

Perfect reconstruction (PR) cosine-modulated filter banks (CMFBs) are of great interests due to their extensive applications that include data compression, denoising, feature detection and extraction, and signal transmultiplexing. Fig. 1 illustrates a typical M-channel maximally decimated parallel filter bank where $H_k(z)$ and $F_k(z)$, $0 \le k \le M-1$, are analysis and synthesis filters. In a CMFB, the impulse response of the analysis and synthesis filters $h_k(n)$ and $f_k(n)$ are cosine-modulated versions of the prototype $h(n)$ [1], i.e.,

$$h_k(n) = 2h(n)\cos\left(\frac{\pi}{2M}(2k+1)\left(n-\frac{N-1}{2}\right)+(-1)^k\frac{\pi}{4}\right),\quad n=0,1,\cdots,N-1$$

$$f_k(n) = 2h(n)\cos\left(\frac{\pi}{2M}(2k+1)\left(n-\frac{N-1}{2}\right)-(-1)^k\frac{\pi}{4}\right),\quad n=0,1,\cdots,N-1,$$

for $k=0,1,\cdots,M-1$, where N is the length of $h(n)$.

The theory and design of PR CMFBs have been studied extensively in the past. In most cases, a PR CMFB design problem is cast to solve following optimization or the analogous,

$$\underset{\boldsymbol{g}\in\mathbb{R}^{mM}}{\text{minimize}}\ \boldsymbol{g}^T\boldsymbol{P}\boldsymbol{g} \tag{1a}$$

$$\text{subject to}\quad \boldsymbol{g}_k^T\boldsymbol{G}_n\boldsymbol{g}_k = \frac{\delta(n)}{2M^2},\quad n=0,1,\cdots,m-1;\quad k=0,1,\cdots,M/2-1, \tag{1b}$$

where m is any positive integer subject to $N=2mM$ throughout this paper, and $\delta(n)$ is given by

$$\delta(n)=\begin{cases}1, & n=0,\\ 0, & \text{otherwise.}\end{cases}$$

For the derivation on $\boldsymbol{g}$, $\boldsymbol{P}$, $\boldsymbol{g}_k$ and $\boldsymbol{G}_n$, please refer to Section 2.1.

Problem (1) is a nonconvex quadratically constrained quadratic programming (QCQP) since the constraints in (1b) are all quadratic equality ones. It is NP-hard in general [2,3]. That is to say, it is very hard to locate its global optimal solution due to the existence of local minima [4]. In some special cases, by exploiting the hidden convexity of this problem, it can be converted into a convex optimization problem [5,6] and then could be solved in convex optimization domain. But in general, only certain relaxation techniques like the semidefinite relaxation [3,7,8], the weighted least squares algorithm [9], etc. are employed to approximately solve this class of problem. And in some cases, the design of PR CMFB

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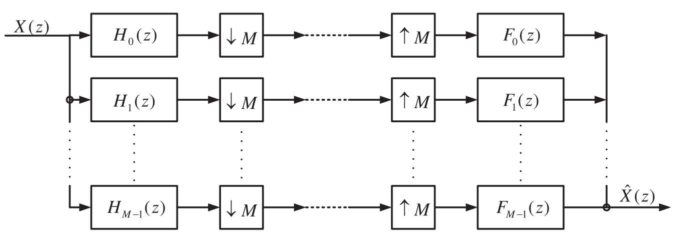

Fig. 1. M-channel maximally decimated filter bank.

is just relaxed to the design of near-perfect reconstruction (NPR) CMFB, which is comparatively easy and many high efficient design methods have been developed for it [8,10–12].

However, an NPR CMFB can not meet the demands of many applications[13,14]. Inspired by this fact, we rethink the quadratically constrained quadratic programming in this paper for the design of PR CMFBs. We design a PR CMFB through consecutive two stages. Firstly, a corresponding NPR CMFB is designed. Then using the designed NPR CMFB as a reference point, we theoretically achieve the goal of designing a PR CMFB by iteratively reducing the constraint violation measurement. Hence the contributions of this paper are: 1) to formulate a PR CMFB design problem as a nonconvex QCQP after rearranging the coefficients of the prototype filter, as well as to an NPR CMFB design problem; 2) to dig out and theoretically show the algebraic relationship between PR conditions and NPR ones; and 3) to propose a two-stage method for the design of PR CMFB in the light of the relationship.

The remainder of this paper is organized as follows. In Section 2, the new method for designing PR CMFBs is proposed in detail. In Section 3, numerical experiments are implemented to validate the proposed method. Finally, conclusions are drawn in Section 4.

2. Design PR CMFBs via QCQP and LS optimization

2.1. Formulation of the design of PR and NPR CMFBs

Consider the M-channel filter bank in Fig. 1. Let $H(z)=\sum_{n=0}^{N-1} h(n)z^{-n}$ with $h(n)=h(N-1-n)$ be the linear-phase prototype filter. In this paper, consider M is even.

In [15], the PR conditions can be stated as[1]

$$E_k(z^{-1})E_k(z)+E_{M+k}(z^{-1})E_{M+k}(z)=\frac{1}{2M^2},\quad k=0,1,\cdots,M/2-1, \tag{2}$$

where $E_k(z)$ are the Type-I $2M$−polyphase components of the prototype filter $h(n)$, i.e.,

$$E_k(z)=\sum_{n=0}^{m-1} h(k+2Mn)z^{-n}. \tag{3}$$

By substituting (3) into (2), the PR conditions that guarantee a distortionless and aliasing-free CMFB can be formulated as (1b) for $n=0,1,\cdots,m-1$; $k=0,1,\cdots,M/2-1$, and

$$\boldsymbol{g}_k=\left[\boldsymbol{h}_k^T\ \ \boldsymbol{h}_{M+k}^T\right]^T,\qquad \boldsymbol{G}_n=\begin{bmatrix}\boldsymbol{D}_n & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{D}_n\end{bmatrix}, \tag{4}$$

where

$$\boldsymbol{h}_k=[h(k)\ h(2M+k)\cdots h(2M(m-1)+k)]^T,$$
$$\boldsymbol{h}_{M+k}=[h(M+k)\ h(3M+k)\cdots h(2Mm-M+k)]^T,$$

$\boldsymbol{D}_n$ is an $m\times m$ matrix with the (p,q)th element given by

$$\boldsymbol{D}_n(p,q)=\begin{cases}1, & \text{if } q-p=n,\\ 0, & \text{otherwise},\end{cases}\qquad p,q=1,2,\cdots,m.$$

Let $\boldsymbol{g}$ be half the set of prototype filter's coefficients resorted as

$$\boldsymbol{g}=[\boldsymbol{g}_0^T\ \boldsymbol{g}_1^T\ \cdots\ \boldsymbol{g}_{M/2-1}^T]^T. \tag{5}$$

Similar to that in [13], we use the stopband edge frequency ω_s and then the stopband energy $\int_{\omega_s}^{\pi}|H(e^{j\omega})|^2 d\omega$ can be written as $\boldsymbol{g}^T\boldsymbol{P}\boldsymbol{g}$. Here $\boldsymbol{P}$ is a positive definite matrix with the (p,q)th element given by

$$\boldsymbol{P}(p,q)=\begin{cases}2\left[\pi-\omega_s-\dfrac{\sin(N-1-2p')\omega_s}{N-1-2p'}\right], & p=q,\\ -2\left[\dfrac{\sin(p'-q')\omega_s}{p'-q'}+\dfrac{\sin(N-1-p'-q')\omega_s}{N-1-p'-q'}\right], & p\neq q\end{cases} \tag{6}$$

for $p,q=0,1,\cdots,N/2-1$ and p', q' indicated by

$$p'=\begin{cases}k+2nM, & \text{if } p=n+2km \text{ and } k+2nM<N/2,\\ N-1-k-2nM, & \text{if } p=n+2km \text{ and } k+2nM\geq N/2,\\ k+(2n+1)M, & \text{if } p=n+(2k+1)m \text{ and } k+(2n+1)M<N/2,\\ N-1-k-(2n+1)M, & \text{if } p=n+(2k+1)m \text{ and } k+(2n+1)M\geq N/2.\end{cases}$$
$$q'=\begin{cases}k+2nM, & \text{if } q=n+2km \text{ and } k+2nM<N/2,\\ N-1-k-2nM, & \text{if } q=n+2km \text{ and } k+2nM\geq N/2,\\ k+(2n+1)M, & \text{if } q=n+(2k+1)m \text{ and } k+(2n+1)M<N/2,\\ N-1-k-(2n+1)M, & \text{if } q=n+(2k+1)m \text{ and } k+(2n+1)M\geq N/2.\end{cases} \tag{7}$$

Thus the design of PR CMFB can be integrally formulated as the combination of (1) and (4)–(7).

Now let's get down to the derivation of an NPR CMFB design problem. In [1], the NPR condition is stated as

$$\sum_{k=0}^{2M-1} H\left(ze^{-jk\pi/M}\right)H\left(z^{-1}e^{jk\pi/M}\right)=1,$$

with j being imaginary unit. It also can be written as [8]

$$\boldsymbol{g}^T\boldsymbol{Q}_n\boldsymbol{g}=\frac{1}{4M}\delta(n),\quad n=0,1,\cdots,m-1, \tag{8}$$

where $\boldsymbol{Q}_n$ is an $(N/2)\times(N/2)$ diagonal block matrix, i.e., $\boldsymbol{Q}_n=\text{diag}(\boldsymbol{G}_n,\cdots,\boldsymbol{G}_n)$. So the NPR CMFB design problem can be formulated as

$$\begin{array}{ll}\underset{\boldsymbol{g}}{\text{minimize}} & \boldsymbol{g}^T\boldsymbol{P}\boldsymbol{g}\\ \text{subject to} & \boldsymbol{g}^T\boldsymbol{Q}_n\boldsymbol{g}=\frac{\delta(n)}{4M},\ \ n=0,1,\cdots,m-1.\end{array} \tag{9}$$

2.2. The relationship between PR and NPR conditions

By the definitions of $\boldsymbol{g}_k$ in (4) and $\boldsymbol{g}$ in (5), following proposition can be given.

Proposition 1. *If the prototype filter with length $N=2mM$ satisfies PR conditions, it holds that:*

[1] Eq. (26) in [15] is multiplied by the scale factor $\frac{1}{M}$ to make (2) coordinate with the derivation of NPR conditions in [1].