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A data driven compressive sensing approach for time-frequency signal enhancement

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ABSTRACT

Signals with the time-varying frequency content are generally well represented in the joint time-frequency domain; however, the most commonly used methods for time-frequency distributions (TFDs) calculation generate unwanted artifacts, making the TFDs interpretation more difficult. This downside can be circumvented by compressive sensing (CS) of the signal ambiguity function (AF), followed by the TFD reconstruction based on the sparsity constraint. The most critical step in this approach is a proper CS-AF area selection, with the CS-AF size and shape being generally chosen experimentally, hence decreasing the overall reliability of the method. In this paper, we propose a method for an automatic data driven CS-AF area selection, which removes the need for the user input. The AF samples picked by the here-proposed algorithm ensure the optimal amount of data for the sparse TFD reconstruction, resulting in higher TFD concentration and faster sparse reconstruction algorithm convergence, as shown on examples of both synthetical and real-life signals.

1. Introduction

In many real-life applications signal frequency content is a key information which needs to be extracted. In order to do this, one can simply use the Fourier transform, but in doing so, signal time attributes are lost. However, by using time-frequency distributions (TFD) one can analyze the evolution of signal energy as a function of both time and frequency, providing additional information about the signal nature. However, if signal has more than one linear frequency modulated (LFM) component, or a non-LFM component, its TFD gets corrupted by highly oscillatory artifacts. Quadratic TFDs (QTFD), most commonly used TFDs in practice, utilise filtering with 2D low-pass filters, which inherently affects time-frequency (TF) localization properties [1]. The need for a trade-off between interference suppression and TF localization has led to a number of TF localization improvement methods, one of which is described in the sequel.

Over the last few years, compressive sensing (CS) has been an important research topic [2–5], with applications in medicine [6,7], geophysics [8,9], communication [10,11], etc. Traditionally, compressive sensing (CS) implies signal sampling with sub-Nyquist frequencies, with samples randomly picked [2–4,6]. However, the samples can be picked to favour specific signal features, while discarding the others [12–14]. This is followed by a signal reconstruction algorithm for solving unconstrained optimization problems (i.e. basis pursuit (BP) [15–22], matching pursuit (MP) [23], orthogonal matching pursuit (OMP) [24,25], etc.). However, the reconstruction algorithm leads to a meaningful result only if the signal is sparse, which means that the signal can be represented in a certain domain with K non-zero coefficients, where $K \ll N_t$ (N_t being the number of signal samples in the time domain). Most signals are non-sparse in the domain of interest, but can become sparse (or approximately sparse) by applying a domain transformation. For example, a sinusoidal signal can be represented with only one sample in the frequency domain. Ideal TFDs are sparse since they are composed of components instantaneous frequency (IF) trajectories, hence CS can be utilized in such a way to include only the signal components samples, while discarding the interference samples; by applying a reconstruction algorithm, resolution loss is minimised [7,8,13,14,25–30].

Current TF signal processing approaches have focused on various optimization algorithms for signal reconstruction, leaving CS area size and shape selection underutilized. The CS area is usually a rectangle, containing approximately N_t samples inside of it [13,26,27]. In this paper, we propose a method for data driven automatic CS area selection. Our goal is to select CS area as large as possible without artifact inclusion, which then increases the signal

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reconstruction algorithm input data amount, hence decreasing its computational requirements.

The paper is organized as follows. Section 2 gives a short introduction to TFDs, while Section 3 describes TFDs as a sparsity inducing signal representation, and introduces the CS area selection algorithm. In Section 4 we compare different optimization algorithms performances based on the measure of reconstructed TFD concentration [31] for the case when the CS area is selected both manually and when it is selected automatically by the proposed method.

2. Quadratic time-frequency distributions

Let us consider an LFM signal $z(t)$ with a time-varying phase $\varphi(t)$, and a slowly varying amplitude $A(t)$ of the form:

$$z(t) = A(t)e^{j\varphi(t)}. \tag{1}$$

Its ideal TFD, $\widehat{\rho}_z(t,\omega)$, is a set of Dirac functions, with a perfect energy localization around the signals instantaneous frequency, $\omega_0(t)$:

$$\widehat{\rho}_z(t,\omega) = A^2(t)\delta(\omega-\omega_0(t)), \tag{2}$$

with $\omega_0(t) = d\varphi(t)/dt$. The ideal TFD in most cases is impossible to accomplish, since practical TFDs are not perfectly localized, and furthermore, they are corrupted by the cross-terms. The Wigner-Ville distribution (WVD), $W_z(t,\omega)$, provides perfect localization for a single LFM component [1]:

$$W_z(t,\omega) = \int_{-\infty}^{\infty} R_z(t,\tau)e^{-j\omega\tau}d\tau, \tag{3}$$

where $R_z(t,\tau)$ is the signal localized autocorrelation function (LAF), defined as:

$$R_z(t,\tau) = z\left(t+\frac{\tau}{2}\right)z^*\left(t-\frac{\tau}{2}\right). \tag{4}$$

However, if the signal has $N_c > 1$ components, its LAF becomes:

$$R_z(t,\tau) = \sum_{i=1}^{N_c} z_i\left(t+\frac{\tau}{2}\right)z_i^*\left(t-\frac{\tau}{2}\right) + \sum_{i=1}^{N_c}\left[z_i\left(t+\frac{\tau}{2}\right)\sum_{\substack{j=1\\ j\neq i}}^{N_c} z_j^*\left(t-\frac{\tau}{2}\right)\right], \tag{5}$$

where the factors under the first sum are the auto-terms, and the remaining factors are the cross-terms. As it can be seen from (5), the cross-terms are mathematical byproduct introduced by LAF's quadratic nature, and will appear midway between each pair of components [1].

In the ambiguity function (AF), $A_z(\nu,\tau)$, defined as:

$$A_z(\nu,\tau) = \int_{-\infty}^{\infty} R_z(t,\tau)e^{-j\nu t}dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_z(t,\omega)e^{-j\nu t}e^{j\omega\tau}dtd\omega, \tag{6}$$

the cross-terms can be filtered out with a 2D low-pass filter, since they are highly oscillatory, and thus located away from the AF plane origin [1]. However, in doing so, the auto-terms get partially filtered out as well, thus reducing TF concentration of the components. This has led to the formulation of QTFD as [1]:

$$\mathcal{A}_z(\nu,\tau) = A_z(\nu,\tau)g(\nu,\tau), \tag{7a}$$

$$\rho_z(t,\omega) = W_z(t,\omega) \underset{t}{*}\underset{\omega}{*} \gamma(t,\omega), \tag{7b}$$

where $g(\nu,\tau)$ and $\gamma(t,\omega)$ are filter functions (also known as kernels), while $\mathcal{A}_z(\nu,\tau)$ and $\rho_z(t,\omega)$ are the filtered AF and TFD, respectively. The symbols $\underset{t}{*}$ and $\underset{\omega}{*}$ denote convolution in time and frequency, respectively.

One of the state-of-the-art TFD is the compact kernel distribution (CKD) defined as [32]:

$$g(\nu,\tau) = \begin{cases} e^{2c}e^{\frac{cD^2}{\nu^2-D^2}}e^{\frac{cE^2}{\tau^2-E^2}} & |\nu|<D,\ |\tau|<E,\\ 0, & \text{otherwise,}\end{cases} \tag{8}$$

where the parameter c defines the shape of the kernel, while the parameters D and E specify the spread of the kernel along the respective AF axis.

Since there is no single best performing kernel for all signals, the need to adaptively construct kernel has arisen. One of the best known methods is the radially Gaussian kernel (RGK) [33], in which the kernel is obtained by solving the optimization problem with the following objective function:

$$g_{\text{opt}}(\nu,\tau) = \arg\max_{g(\nu,\tau)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|A_z(\nu,\tau)g(\nu,\tau)|^2d\tau d\nu, \tag{9}$$

with the constraints which ensure that the kernel has properties of the low-pass filter. Since cross-terms do not contain signal energy, maximizing (9) ensures that the auto-term energy is transferred from the AF to the resulting TFD, without attenuation, while avoiding the cross-terms.

3. Time-frequency distribution as a sparsity inducing domain

3.1. Compressed sensed ambiguity function

As mention earlier, ideal TFDs are inherently sparse, since they are composed of the components IFs, thus requiring only $N_cN_t < <N_\omega N_t$ samples, where N_c, N_t, and N_ω are the number of components, the number of time instances, and the number of frequency bins, respectively. The CS-AF, $A'_z(\nu,\tau)$ is formulated as:

$$A'_z(\nu,\tau) = \phi(\nu,\tau)\odot A_z(\nu,\tau), \tag{10}$$

where the operator $\odot$ denotes element-by-element matrix multiplication, and $\phi(\nu,\tau)$ is the sensing matrix which defines $N'_\nu \times N'_\tau$ area Ω around the AF plane origin:

$$\phi(\nu,\tau) = \begin{cases} 1, & (\nu,\tau)\in\Omega,\\ 0, & \text{otherwise.}\end{cases} \tag{11}$$

The sensing matrix defined in this way discards highly oscillatory cross-terms located away from the domain origin, while preserving the auto-terms located closer to the origin.

In the standard CS notation, matrix multiplication is commonly used to define connection between the observation and solution matrices, thus from (6) and (10):

$$A'_z(\nu,\tau) = \psi\cdot\vartheta_z(t,\omega), \tag{12}$$

where ψ is a domain transformation matrix, representing a two-dimensional Fourier transform, and $\vartheta_z(t,\omega)$ is a sparse TFD reconstructed from the CS-AF. However, in most practical realizations, ψ and its inverse are implemented as functions, since matrix free algorithms are significantly faster and require less memory space [17,20].

Since $A'_z(\nu,\tau)$ has cardinality $\text{card}(A'_z(\nu,\tau)) = N'_\nu\cdot N'_\tau$ samples, and $\vartheta_z(t,\omega)$ has cardinality of $\text{card}(\vartheta_z(t,\omega)) = N_t\cdot N_\omega$ samples, where $\text{card}(A'_z(\nu,\tau)) \ll \text{card}(\vartheta_z(t,\omega))$, the system is underdetermined, thus $\vartheta_z(t,\omega)$ can have an infinite number of possible solutions, and the goal of the reconstruction algorithm is to find the optimal solution to:

$$\vartheta_z(t,\omega) = \psi^H\cdot A'_z(\nu,\tau) = \psi^H\cdot\phi(\nu,\tau)\odot A_z(\nu,\tau), \tag{13}$$