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Topology optimization utilizing inverse motion based form finding

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Abstract

Topology optimization at finite strain setting using the concept of inverse motion based form finding is introduced. This novel procedure allows boundary conditions and shape of the structure in the operating, deformed, state to be prescribed. The outcome of the optimization algorithm will be the shape of the undeformed structure, i.e. the state in which the structure should be manufactured. The objective of the optimization considered is to find the stiffest structure for a given amount of material. The problem is regularized using a Helmholtz filter which is formulated in the deformed configuration. Both the elastic boundary value problem and the partial differential equation associated with the Helmholtz filter are solved using the finite element method. The optimization problem is solved using a sequence of convex separable approximations. The paper is closed by 2D as well as 3D numerical examples that clearly illustrates that the method is able to find optimal solutions for inverse motion finite strain topology optimization problems.

Keywords: Topology optimization; Inverse motion form finding; Finite strains

1. Introduction

Structural optimization has evolved rapidly over the past decades and the methods are today widely used within industry while still being subject to intense research. One branch of structural optimization is concerned with finding an optimal design within a given design domain, i.e. topology optimization, cf. [1,2] or [3] for overviews of topology optimization and its applications. The objective of an optimization can for instance be to minimize stresses, displacements, eigenvalues or stiffness. Due to the simple numerical treatment associated with the stiffness, this objective is frequently considered in the literature. As continuum based optimization where the goal is to find a distinct black and white design is ill-posed, a length scale needs to be introduced into the formulation. This regularization can be accomplished via e.g. filtering, level set cf. e.g. [4] and phase-field approaches [5–7] where the filtering procedure by far is the most frequently employed procedure. Filtering can be directly applied on the density (cf. [8]) or on the sensitivities (cf. [9]) or via the solution of a partial differential equation (PDE), i.e. a Helmholtz filter, cf. [10].

A fast and robust solution procedure for structural optimization problems that contain a large number of design variables can be established by formulating a sequence of separable convex sub problems that approximates the

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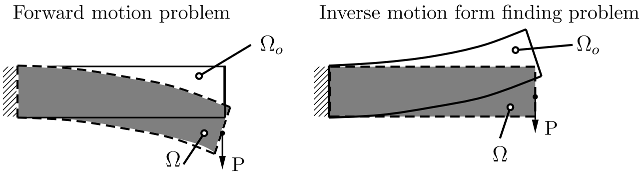

Fig. 1. Illustration of forward motion and inverse motion form finding formulations. Solid line indicates the reference configuration, Ω_o, and the dashed line indicates the deformed configuration, Ω. The solid lines indicate the shape obtained from the solution procedure. The rectangular box is taken as the computational domain (or design domain) and therefore known and fixed in both strategies.

original problem in the neighborhood of the current design. Separable convex sub problems can efficiently be solved using e.g. CONvex LINearization (CONLIN), cf. [11] and the Method of Moving Asymptotes (MMA), cf. [12]. An efficient numerical solution strategy is given by the Solid Isotropic Material Penalization (SIMP). In the SIMP approach infinitesimal strains are typically considered in conjunction with linear isotropic elasticity.

Research on finite strain based topology optimization do exist (cf. [13–19]) but is relatively limited. One difficulty associated with finite strain topology optimization formulations is that excessive deformations may occur in regions of low density and thereby causing convergence problems due to inverting elements in the solution process of the elastic boundary value problem, cf. [20–22]. In addition to low-density problems, stability issues such as snap-through might exist, cf. [23,17]. Another problem associated with finite strain based stiffness topology optimization is related to the definition of the stiffness. For infinitesimal strains the definition of the stiffness, or its inverse the compliance, is well defined. For finite strains, however, several potential candidates for defining the stiffness are available, e.g. the end-displacement and potential energy. Yet another problem when considering finite strains is related to the definition of the elastic relation which for infinitesimal strains is straightforward. When finite strains are considered, several strain energy potential functions are available (cf. e.g. [18]). The importance of using a polyconvex strain energy potential (cf. [24]) has been shown by [15,16,19].

In the analysis of nonlinear elastic structures, typically, a load is applied on the boundary and the response in terms of e.g. displacements and stresses are calculated. The original, unloaded reference configuration is known before the load is applied and the deformed state will be part of the outcome of the analysis. This solution strategy will subsequently be referred to as the forward motion problem. When designing structures where the shape in the deformed state is of major importance the usefulness of the forward motion problem formulation is limited since the deformed state, the service state, cannot be prescribed. To overcome this problem the inverse motion form finding problem formulation can be utilized. In this approach the deformed configuration is taken as known and the reference configuration will be the outcome of the analysis, cf. [25–28]. By using the proposed approach the final outer shape and boundary conditions can exactly be prescribed at the deformed state. This is in contrast to other procedures where the reference state is fixed and the analysis will provide the deformed state. It is foreseen that the new method will be of importance where the outer contour of the structure is interacting with its surrounding.

The inverse motion procedure is numerically similar to the classical forward motion problem and the major difference is that the inverse deformation gradient is driving the problem instead of the deformation gradient associated with the forward motion problem. An illustration of the forward motion and inverse motion form finding problem is given in Fig. 1.

In the present paper a finite strain topology optimization scheme based on the concept of inverse motion form finding analysis will be presented. The objective in the optimization will be the potential which in the limit of infinitesimal strains coincides with the stiffness. The regularization required to avoid a micro-structure being formed will be performed by the Helmholtz filter as proposed in [10]. The paper will be closed by 2D and 3D numerical examples that clearly demonstrates the possibility of the proposed numerical procedure.

2. Problem definition

The present work is concerned with optimization of geometrically non-linear elastic structures and for that sake the kinematics that defines the motion of a body is introduced. At time t_o, the structure occupies the configuration Ω_o which subsequently is referred to as the reference configuration. At an instant $t > t_o$, the structure has been deformed such that it occupies Ω which is referred to as the deformed, current, configuration. The boundaries to Ω and Ω_o are denoted $\partial\Omega$ and $\partial\Omega_o$, and the outward unit normals to Ω and Ω_o are denoted $\boldsymbol{n}$ and $\boldsymbol{N}$. A particle which at time t_o is