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Even/odd decomposition made sparse: A fingerprint to hidden patterns

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A B S T R A C T

The very fundamental operation of even/odd decomposition is at the core of some of the simplest information representation and signal processing tasks. So far most of its use has been for rearranging data to provide fast implementations of various types of transforms (Fourier, DCT, ...) or for achieving elementary data transformation, such as the Walsh–Hadamard transform. This work proposes to look into the decomposition framework to obtain a richer perspective. In the context of an iterated even/odd decomposition, it is possible to pinpoint intermediate layered levels of symmetries which cannot be easily captured in the original data. In addition this determines a hierarchical fingerprinting for any sort of continuous finite support analog signal or for any discrete-time sequence which may turn out useful in several recognition or categorization tasks. It also may help to achieve sparsity within a natural hierarchical framework, which could be easily extended for many other types of orthogonal transformations. This paper also suggests a global measure of the energy imbalance across the hierarchy of the decomposition to capture the overall fingerprinting of this interpretation.

1. Introduction

The need to find particular signal characteristics, e.g. to make classification and/or labeling possible, is inherent in many signal processing applications. These include but are not limited to tasks such as event classification, anomaly detection, denoising, ... [3,14,16]. Such a broad class of applications adopts the most diverse technical solutions. It is however possible to enumerate a number of common approaches to such problems, all based on the search for possible hidden patterns in the data, for example the presence of (locally) periodic signals. In many cases, specific patterns are directly looked for thanks to pattern matching techniques [1] or indirectly through correlation-based measures [2]. Other possible approaches exploit specific stochastic properties found in natural data [11]. Feature extraction is a commonly found intermediate step, that is applied either in the original data domain or in a transformed one.

A particular approach, which is not always part of the aforementioned techniques, consists in finding symmetries of some kind that arise naturally for many classes of signals. Interest in symmetry detection exists for many different communities and it is aimed at various signal modalities. Symmetries can be either local or global in nature, and the search methods for these symmetry classes can be quite different (see [13] and [18]). It is well known that exploiting signals' inherent symmetries is an effective way to model the source, which may turn out useful for e.g. information compression [17]. Such alternative information descriptions have been widely adopted, since they allow to condense (sparsify) important properties of the original signal. Often that is why a reversible transformation of the original data is applied to reach this more compact description. Good examples of such transformations are omnipresent: see Fourier and multiresolution transforms [14,15]. Non linear alternatives exist such as the iterative function system (IFS) paradigm for computing fractal dimensions or near-by signal regeneration [10].

One of the simplest decomposition proposed in signal processing, which is by its very definition constructed upon the signals symmetries, is the even/odd decomposition [15]. It turns out that, given the resulting intuitive geometrical interpretation and the parity preservation of the Fourier transform, even/odd decomposition is quite common in signal processing. It becomes then a natural proposition to try iterating this process to each half of the even/odd parts, which are necessary for reconstruction. The same geometrical interpretation can thus be preserved over the resulting decomposition tree. This in turn gives a peculiar characterization of the signal that is based on how its decomposition tree is shaped.

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Therefore, this paper studies how to perform an iterative even/odd decomposition of 1–D signals around their midpoint, which in addition allows for a fast implementation. Such process is possible for both continuous and discrete-time signals and involves only very simple operations at each stage, without increasing the interval support or the number of samples necessary to represent the original signal. It will be shown that the recursive application of the even/odd decomposition for discrete sequences provides results identical to the application of a radix–2 implementation of the Walsh–Hadamard discrete transform (WHT) [5]. On another hand, the iterative nature of the transform allows for a deeper analysis of hidden symmetric patterns in the data during the computation. Such patterns do not correspond to local symmetries but are instead indicative of an even/odd relation between parts of the signal existing at a particular level of the decomposition tree, thus permitting to make decisions, such as to arrest the resulting decomposition tree at an earlier stage, without significantly impairing the quality of the representation. This approach therefore ensures both a fast implementation and an efficient way to detect these peculiar symmetric relations, leading thus to a naturally sparse representation of the decomposition tree. To prove how sparsity in the decomposition tree is an useful signal characterization, a tree sparsity measure is employed to classify broad 1–D signal types.

The rest of the presentation is organized as follows. Section 2 fixes some notation and presents some preliminary processing background by recapping briefly the even/odd decomposition of signals, along with how such operation motivates the considerations put forward in this paper. Section 3 provides a description of the iterative application of the even/odd decomposition to obtain a decomposition tree in the case of continuous time signals. The discussion is extended to the discrete time case in Section 4, leaving the resolution of some caveats that such domain causes till Section 6. The paper follows with some experimental simulation results described in Section 5 and ends by drawing the conclusions in Section 7.

1.1. Contributions

This paper introduces a decomposition tree for finite energy signals using the basic even/odd decomposition as the core decomposition step. It provides a blueprint for its fast implementation for finite, discrete sequences through the recursive application of a “butterfly”–like computation, similar to that employed in the fast computation of the Walsh–Hadamard Transform. As opposed to WHT, the iterative nature of the proposed decomposition allows to analyze every step of the process (the decomposition level), for example to extract features on the tree nodes. This paper is in particular focused on detecting sparsity in the decomposition tree as it is built from the root (the original signal) up to the leaves. Sparsity is measured during the iterative generation of intermediate results rather than relying on the L_0 norm of the leaves of the decomposition tree. Proving how such sparsity can provide a discriminating feature across different data types, it is suggested how such decomposition tree can find a particular type of symmetry in the data, that is not directly connected to local or global symmetries of the whole signal.

2. Motivation and background

An iterative decomposition process can be undertaken based on the well–known even/odd decomposition basic signal manipulation, sometimes also referred to as the parity decomposition. The even/odd decomposition of a given energy signal $x(t) \in \mathcal{L}^2(\mathbb{R})$ states that $x(t)$ can be expressed as the sum of its even and odd parts, respectively $x_e(t)$ and $x_o(t)$, given by:

$$x_e(t) = \frac{x(t)+x(-t)}{2}; \quad x_o(t) = \frac{x(t)-x(-t)}{2};$$
$$x(t) = x_e(t) + x_o(t) \tag{1}$$

The even signal is such that $x_e(t) = x_e(-t)$; the odd signal is such that $x_o(t) = -x_o(-t)$. Since $\mathcal{L}^2(\mathbb{R})$ is a Hilbert space, with inner product $<x(t), y(t)>= \int_{\mathbb{R}} x(t)y^*(t)dt$, such decomposition is possible $\forall\, x(t)$ and represents the vector $x(t)$ as the sum of two orthogonal vectors since the inner product $< x_e(t), x_o(t) >$ is obviously 0.

The energy E is defined as the squared Euclidean norm of the signal $x(t)$ and it is easy to see that:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x_e(t) + x_o(t)|^2 dt = \tag{2}$$
$$= \int_{-\infty}^{\infty} |x_e(t)|^2 dt + \int_{-\infty}^{\infty} |x_o(t)|^2 dt = E_e + E_o$$

where E_e and E_o are the energy of the even and odd parts respectively. The last step exploits the orthogonality of $x_e(t)$ w.r.t. $x_o(t)$.

The motivation under our interest in this elementary operation is its ability to readily detect inherent symmetries in the data. In fact, if the original signal $x(t)$ is of an inherently even (resp. odd) shape, the most part of its energy will be carried by its even (resp. odd) component. For example, in Fig. 1 the latter case applies: the odd part carries around 70% of the total energy, or more than twice as much as that of the even part.

This characteristic can be generally useful for a number of tasks in signal processing, e.g. it favors a compact representation of the original signal. In fact, if the signal is reconstructed not by summing both parts but just by retaining the one which carries the most energy, the signal can be represented without introducing too much distortion. Obviously, in general for a given signal it is unlikely that such a condition holds after a single decomposition step, unless the signal possesses a very obvious symmetric/antisymmetric shape. For example, in Fig. 1 one could hardly imagine to represent $x(t)$ using only $x_o(t)$.

For finite support signals, let us now propose to iterate the analysis, by applying the decomposition on the resulting informative part of the even and odd signals (i.e. their causal part which is recentered around the origin), thus constructing a decomposition tree. As the signals are decomposed again and again through Eq. (1) into pairs of orthogonal vectors, they can be analyzed in turn to exploit the symmetry content description which is inherent in the energy they carry. For example, to continue with the example above regarding the compact representation of a given signal, after a certain number of decompositions it can happen that (at least) one of the constituting signal can be safely discarded because it has energy below a certain tolerance threshold.

Of course, there are many possible strategies to handle the signals resulting from the iterative application of the even/odd decomposition, i.e. the decomposition tree nodes, depending on what is the intended objective. For example, for strongly symmetric data it could happen that, ending the process after a certain number of decompositions, just a small fraction of the tree leaves carry almost all of the energy of the original signal, thus retaining just them to reconstruct the signal leads to a compact representation of it. Hence, we can set such a threshold B, which can be possibly adapted to the decomposition level l being considered. If some of the leaves or nodes carry less energy than B they are omitted from the reconstruction. If one of the leaves is missing because it has been discarded, only the surviving companion leaf is used to reconstruct the parent tree node.

On the other hand, detecting an energy imbalance in some nodes of the decomposition tree corresponds to a hidden symmetric relation which is present at the corresponding level of the de-