Signal Processing 140 (2017) 149–160

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Joint DOA estimation and source number detection for arrays with arbitrary geometry

Falamarz Izedi, Mahmood Karimi*, Mostafa Derakhtian

School of Electrical and Computer Engineering, Shiraz University, Shiraz, Iran

ARTICLE INFO

Article history:
Received 22 October 2016
Revised 20 April 2017
Accepted 6 May 2017
Available online 16 May 2017

Keywords:
DOA estimation
Source number detection
Array processing
Nonuniform array

ABSTRACT

In this paper, we focus on the problem of joint direction-of-arrival (DOA) estimation and source number detection for an array of sensors. We propose a CLEAN-based sequential algorithm that uses a sequential hypothesis testing procedure. This method can be employed for any array with an arbitrary geometry. We also manage to reduce the computational complexity of the proposed method. Additionally, we derive an analytical performance bound for the source number detection algorithm. Our simulation results show that the proposed method achieves an appropriate performance even for low SNR or small number of snapshots. It is shown that the proposed method is applicable to both correlated and uncorrelated sources. Unlike popular methods, the algorithm does not require to know the number of sources for the DOA estimation and is able to estimate the number and the DOA of sources jointly.

1. Introduction

Array signal processing has applications in areas such as radar, sonar, wireless communication, radio astronomy, seismology, acoustics, and medical imaging [1,2]. Two of the most important problems in the array signal processing are the direction-of-arrival (DOA) estimation and source number detection. Due to the cost, available space and system performance in practical scenarios, it tends to be unsuitable to restrict the array geometry to a certain class. Therefore, the methods that can be applied to arbitrary array geometries are in great demand [3,4].

For the DOA estimation problem, the multiple signal classification (MUSIC) algorithm [5] is one of the well-known subspace-based methods. This algorithm can be applied to any type of array geometry. A version of this algorithm, which is referred to as root-MUSIC, utilizes polynomial rooting [6]. However, this method is only applicable to the uniform linear array (ULA). In [7] a method is presented to extend the application of the root-MUSIC to the nonuniform linear array case. Fourier-domain root-MUSIC is another method which develops the root-MUSIC algorithm for DOA estimation in the sensor arrays of an arbitrary geometry [8]. Another popular technique for DOA estimation is the estimation of signal parameters via rotational invariance techniques (ESPRIT) [9]. In the ESPRIT method, the array geometry is required to be shift invariant which limits the application of this method to a certain class of arrays. Array interpolation is a DOA estimation technique employed for nonuniform arrays [10–12]. This method involves applying transformation to the received signal in order to obtain an interpolation of the signal over a virtual ULA. Manifold separation technique [13] models the received waveform by means of an orthogonal expansion and approximates the true array steering vector as the product of a matrix that depends only on the array parameters and a Vandermond vector which depends merely on the angle of arrival.

Some other DOA estimation methods are based on a technique called CLEAN [14]. CLEAN is developed initially for radio astronomy. The main idea of CLEAN is to remove the strongest signals from the observed data successively. In [15], a simple version of CLEAN method is applied to the DOA estimation problem. In this simple version, the cancellation of signal components is not perfect and as a result, the performance of DOA estimation degrades. In [3], a method is proposed for DOA estimation in nonuniform linear arrays. This method is a combination of CLEAN, Root-MUSIC and Toeplitz Completion techniques. However, this method needs an initial estimate of DOA of sources and the number of sources must be known.

A popular sequential search technique for finding a sparse solution is orthogonal matching pursuit (OMP) [16]. This method can be utilized for DOA estimation.

Source signals are correlated on condition that multipath reflections are present. When the source signals are correlated, the signal covariance matrix may become rank-deficient and many of the

* Corresponding author.
E-mail addresses: izedi@shirazu.ac.ir (F. Izedi), karimi@shirazu.ac.ir (M. Karimi), derakhtian@shirazu.ac.ir (M. Derakhtian).

http://dx.doi.org/10.1016/j.sigpro.2017.05.008
0165-1684/

mentioned methods fail to find the source DOA [3]. Certain methods such as forward-backward spatial smoothing (FBSS) [18] can be used to improve the rank. The FBSS is one of the best methods to deal with the correlated sources, particularly when ULAs are considered [3]. The disadvantage of this technique is that it uses the subarrays and therefore the resulting covariance matrix size is less than the original covariance matrix size. It must also be pointed out that the FBSS cannot be used directly for an array with an arbitrary geometry.

Regarding the source number detection problem, the minimum description length (MDL) method [19] is one of the most successful methods. Another popular method is Akaike information criterion (AIC) [20]. These methods share common origins in the information-theoretic and Bayesian formulation of the general model selection problem [19]. The MDL and AIC are known to suffer in the detection performance for the small number of snapshots.

In this paper, we propose a CLEAN-based algorithm by solving a sequential generalized likelihood ratio test (GLRT) to estimate both DOA and the number of sources. This method can be used for arrays with an arbitrary geometry. Moreover, we reduce the computational complexity of the proposed method. An analytical performance bound for source number detection is also derived. It has been illustrated that the proposed technique has an appropriate performance in the case of small snapshots for two closely spaced targets. The proposed method is applicable to both correlated and uncorrelated sources. It is noteworthy that the proposed method is a CLEAN-based algorithm that is developed through an statistical approach by solving a sequential generalized likelihood ratio test. It is also worth mentioning that although some versions of CLEAN exist in the literature, there appear to be very few statistical studies about this method [15]. To the best of our knowledge, no other method in the literature looks at the CLEAN algorithm from the GLRT point of view.

The remainder of this paper is organized as follows. Section 2 describes the signal model. The proposed method for the joint DOA estimation and source number detection is presented in Section 3. Section 4 is presented to reduce the computational complexity of the proposed method. The performance bound of the proposed source number detection algorithm, is analyzed in Section 5. Simulation results, presented in Section 6, explore the validation of the theoretical results. Finally, Section 7 concludes the paper.

Notation: Matrices are denoted by the upper boldface letters and vectors by the lower boldface letters. $|x|$ shows the absolute value of x and $\|\mathbf{x}\|$ stands for the Euclidean norm. The superscript ^{H} denotes the Hermitian of a matrix or a vector. $\Pr(\cdot)$ stands for the probability. Furthermore, we use the notation $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$ to indicate the complex normal distribution with the mean $\boldsymbol{\mu}$ and the covariance matrix $\mathbf{C}$. Finally, $\mathbf{I}$ is the identity matrix.

2. The signal model

Consider an array of M omnidirectional sensors with nonuniform spacing in the xy plane. Assume that this array receives signals from $L (L < M)$ narrowband far-field sources with the unknown DOAs, $\boldsymbol{\theta} \triangleq [\theta_1, \ldots, \theta_L]^T$. We intend to estimate the number of sources and their DOAs. The $M \times 1$ array output vector at the nth snapshot can be modeled as [1]:

$$\mathbf{x}(n) = \mathbf{A}_L(\boldsymbol{\theta})\mathbf{s}_L(n) + \mathbf{w}(n), \tag{1}$$

where $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_L]^T$ is the $L \times 1$ vector of signal DOAs. $\mathbf{A}_L(\boldsymbol{\theta}) \triangleq [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_L)]$ is the $M \times L$ signal steering matrix and $\mathbf{a}(\theta_i)|_{i=1,\ldots,L}$ is the $M \times 1$ steering vector of the ith source which can be expressed as

$$\mathbf{a}(\theta_i) = \Big[\exp\Big(j\frac{2\pi}{\lambda}(x_1 \sin\theta_i + y_1 \cos\theta_i)\Big), \ldots, \times \exp\Big(j\frac{2\pi}{\lambda}(x_M \sin\theta_i + y_M \cos\theta_i)\Big)\Big], \quad i = 1, \ldots, L, \tag{2}$$

where λ is the wavelength of the signal and $\{x_m, y_m\}|_{m=1,\ldots,M}$ are the coordinates of the mth array sensor. $\mathbf{s}_L(n) \triangleq [s_1(n), \ldots, s_L(n)]^T$ is the $L \times 1$ vector of the signal waveforms and $\mathbf{w}(n) \triangleq [w_1(n), \ldots, w_M(n)]^T$ is the $M \times 1$ vector of the complex Gaussian sensor noises with the zero mean and covariance matrix $\sigma^2\mathbf{I}$. Hereafter, for simplicity, we show $\mathbf{A}_L(\boldsymbol{\theta})$ by $\mathbf{A}_L$ and $\mathbf{a}(\theta_i)$ by $\mathbf{a}_i$.

3. The proposed method

In order to estimate the number of sources, we propose a sequential algorithm which results from the solution of a sequential composite hypothesis testing problem. Solving this problem at the ℓth stage will determine whether there is a source in the direction of θ_ℓ or not. The presence or absence of this source must be determined in the presence of the other sources in the unknown direction of $\theta_1, \theta_2, \ldots, \theta_{\ell-1}$. The hypothesis testing problem at the ℓth stage can be modeled as

$$\begin{cases} \mathcal{H}_0^{(\ell)} : \mathbf{x}(n) = \mathbf{A}_{\ell-1}\mathbf{s}_{\ell-1}(n) + \mathbf{w}(n), & n = 1, \ldots, N; \\ \quad \ell = 1, \ldots, L \\ \mathcal{H}_1^{(\ell)} : \mathbf{x}(n) = \mathbf{A}_{\ell-1}\mathbf{s}_{\ell-1}(n) + \mathbf{a}_\ell s_\ell(n) + \mathbf{w}(n), & n = 1, \ldots, N; \\ \quad \ell = 1, \ldots, L, \end{cases} \tag{3}$$

where $\mathbf{A}_{\ell-1}$ is the $M \times (\ell-1)$ steering matrix of $\ell-1$ sources; more explicitly,

$$\mathbf{A}_{\ell-1} \triangleq [\mathbf{a}_1, \ldots, \mathbf{a}_{\ell-1}]. \tag{4}$$

$\mathbf{s}_{\ell-1}(n) \triangleq [s_1(n), \ldots, s_{\ell-1}(n)]^T$ is the $(\ell-1) \times 1$ vector of the signal waveforms and $s_\ell(n)$ is the scalar signal of the ℓth source. This is a composite hypothesis problem with the unknown parameters θ_i and $s_i(n)$ for $i = 1, \ldots, \ell$. The noise variance is assumed to be known. We employ the GLR test to solve the composite hypothesis problem. The details of the derivation of this GLR-based detector is presented in Appendix A. The likelihood ratio is

$$\mathcal{L}_\ell(\mathbf{x}) = \max_\theta \sum_{n=1}^{N} \frac{|\mathbf{a}^H(\theta)\mathbf{P}_{\ell-1}^\perp \mathbf{x}(n)|^2}{\mathbf{a}^H(\theta)\mathbf{P}_{\ell-1}^\perp \mathbf{a}(\theta)} = \sum_{n=1}^{N} \frac{|\mathbf{a}_\ell{}^H \mathbf{P}_{\ell-1}^\perp \mathbf{x}(n)|^2}{\mathbf{a}_\ell{}^H \mathbf{P}_{\ell-1}^\perp \mathbf{a}_\ell} \underset{\mathcal{H}_0^{(\ell)}}{\overset{\mathcal{H}_1^{(\ell)}}{\gtrless}} \eta_\ell, \tag{5}$$

where η_ℓ is the ℓth stage threshold which is determined based on the ℓth stage false alarm probability, $\mathbf{a}_\ell$ is the $M \times 1$ steering vector corresponding to the estimated direction θ_ℓ, and $\mathbf{P}_{\ell-1}^\perp$ is a matrix that is orthogonal to the $\mathbf{A}_{\ell-1}$, and is given by

$$\mathbf{P}_{\ell-1}^\perp = \begin{cases} \mathbf{I} - \mathbf{A}_{\ell-1}\left(\mathbf{A}_{\ell-1}^H \mathbf{A}_{\ell-1}\right)^{-1}\mathbf{A}_{\ell-1}^H & \ell > 1 \\ \mathbf{I} & \ell = 1, \end{cases} \tag{6}$$

and as a result $\mathcal{L}_1(\mathbf{x}) = \sum_{n=1}^{N} \frac{|\mathbf{a}_1{}^H \mathbf{x}(n)|^2}{M}$. Using (5), our CLEAN-based algorithm for joint DOA estimation and source enumeration can be summarized as follows:

Step 1 Compute $\mathcal{L}_1(\mathbf{x})$, and then compare it with the predetermined threshold value, η_1. If this value is not greater than the threshold value, stop the algorithm and $L = 0$. Otherwise $\hat{\theta}_1 = \arg\max_\theta \sum_{n=1}^{N} \frac{|\mathbf{a}^H(\theta)\mathbf{x}(n)|^2}{M}$ and go to the step 2.

Step 2 For $k = 2, 3, \ldots,$ perform the following procedure:

1) Define $\mathbf{A}_{k-1}$ as $\mathbf{A}_{k-1} \triangleq \left[\mathbf{a}_1, \ldots, \mathbf{a}_{k-1}\right]$. Where $\mathbf{a}_{k-1}$ is the steering vector in the direction of $\hat{\theta}_{k-1}$.

2) Compute $\mathbf{P}_{k-1}^\perp$ for $\mathbf{A}_{k-1}$ using (6) and then calculate $\mathcal{L}_k(\mathbf{x})$ from (5).