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Short communication

Off-grid DOA estimation with nonconvex regularization via joint sparse representation

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ABSTRACT

In this paper, we address the problem of direction-of-arrival (DOA) estimation using sparse representation. As the performance of on-grid DOA estimation methods will degrade when the unknown DOAs are not on the angular grids, we consider the off-grid model via Taylor series expansion, but dictionary mismatch is introduced. The resulting problem is nonconvex with respect to the sparse signal and perturbation matrix. We develop a novel objective function regularized by the nonconvex sparsity-inducing penalty for off-grid DOA estimation, which is jointly convex with respect to the sparse signal and perturbation matrix. Then alternating minimization is applied to tackle this joint sparse representation of the signal recovery and perturbation matrix. Numerical examples are conducted to verify the effectiveness of the proposed method, which achieves more accurate DOA estimation performance and faster implementation than the conventional sparsity-aware and state-of-the-art off-grid schemes.

1. Introduction

Direction-of-arrival (DOA) estimation has been extensively studied over the past few decades because of its fundamental role in many signal processing areas ranging from multiple-input multiple-output radar, mobile and wireless communications, channel estimation and sonar to acoustic tracking [1–3].

Recently, sparse representation has attracted increasing interest in statistical signal analysis and parameter estimation. In [4], the concept of sparse representation is extended to address the problem of DOA estimation problem and ℓ_1-SVD algorithm is proposed to reduce the dimension of observations via singular value decomposition (SVD), which can achieve super-resolution performance. A reweighted ℓ_1 norm penalty algorithm [5] exploits the coefficients of the reduced dimension Capon spatial spectrum in constructing the weight matrix to enforce the sparsity of solution, which involves a high computational burden. The methods mentioned above have shown improvements in DOA estimation, but most of them are based on on-grid DOA ℓ_1 norm constrained minimization. Since in practice the unknown DOAs are not always exactly on the sampling grids, their DOA estimation performance will degrade due to errors caused by the mismatches.

To circumvent this issue, off-grid DOA estimation methods have been developed [6–12]. In [6], a gridless sparse approach via reweighted atomic norm minimization is proposed for off-grid DOA estimation. In [7,8], alternating minimization is exploited to solve for sparse signal and dictionary mismatch simultaneously, but it suffers from slow convergence. A noise subspace fitting-based off-grid DOA estimation method is derived in [9] using second-order Taylor approximation to achieve higher modeling accuracy. In [10], an analytical performance bound on joint sparse recovery is given and a fast iterative shrinkage-threshold algorithm is implemented to tackle joint sparse recovery with structured dictionary mismatches. In [11], co-prime arrays are considered to increase degrees of freedom for the grid mismatch and sample covariance matrix is utilized to reduce the effect of noise variance. In [12], a computationally efficient root sparse Bayesian learning (RSBL) method is proposed to eliminate the modeling error when using coarse grid.

Compared with the convex function regularized by least squares problem, it has been demonstrated that utilizing nonconvex functions, such as smoothed ℓ_0 quasi-norm [13], ℓ_p quasi-norm [14] and weak convexity [15], can achieve better sparse signal recovery. In this paper, we develop a novel objective function regularized by the nonconvex sparsity-inducing penalty for off-grid DOA estimation. Our motivation is twofold: (i) to overcome

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the limitation of the conventional sparsity-based DOA estimation methods that the unknown angles belong to predefined discrete angular grids; and (ii) a proper nonconvex regularization is able to achieve better performance compared with convex relaxation employing the ℓ_1 norm function. In this study, we first introduce the off-grid model into DOA estimation via first-order Taylor series expansion, which is equivalent to the dictionary mismatch, and then devise an objective function regularized by the nonconvex sparsity-inducing penalty with the least absolute shrinkage and selection operator (LASSO) [16]. The resulting objective function is jointly convex with respect to the sparse signal and perturbation matrix. We follow the rationale of alternating minimization to obtain the sparse signal by alternating direction method of multipliers (ADMM) [17] with incorporating the proximity operator for a fixed perturbation matrix, then update perturbation matrix via fixing the sparse signal and so on. Our results demonstrate that the proposed method outperforms the conventional sparsity-aware and state-of-the-art off-grid schemes.

The rest of this paper is organized as follows. In Section 2, the problem of DOA estimation using sparse representation is formulated. Section 3 introduces the off-grid model and presents our DOA estimation method. In Section 4, numerical examples are conducted to evaluate the performance of the proposed algorithm. Section 5 concludes this paper.

Notation: Lowercase bold-face and uppercase bold-face letters represent vectors and matrices, respectively. $(\cdot)^{\dagger}$, $(\cdot)^T$ and $(\cdot)^H$ are pseudo-inverse, transpose and conjugate transpose operators, respectively. vec$(\cdot)$ denotes the vectorization operator which stacks a matrix column by column. diag$(\cdot)$ is a diagonal matrix composed of the elements of a column vector. $\otimes$ denotes the Kronecker product operator. $||\cdot||_1$, $||\cdot||_2$ and $||\cdot||_F$ denote the ℓ_1 norm, ℓ_2 norm and Frobenius norm, respectively. $\Re$ and $\Im$ take the real and imaginary parts of a complex variable, respectively. $\mathbf{I}_K$ denotes the $K \times K$ identity matrix.

2. Problem statement

2.1. Signal model

Consider a uniform linear array (ULA) equipped with M sensors. The inter-element spacing is half-wavelength. The origin is set at the middle point of the ULA. Assume that K narrowband signals from the far-field impinge onto the ULA from unknown and distinct angles of $\theta_1, \ldots, \theta_K$. The ULA response at the kth target can be expressed as

$$\mathbf{a}(\theta_k) = [e^{-j\pi \frac{(M-1)}{2}\cos(\theta_k)}, \ldots, e^{j\pi \frac{(M-1)}{2}\cos(\theta_k)}]^T \tag{1}$$

The $M \times 1$ observation vector is:

$$\mathbf{y}_t = \mathbf{A}(\theta)\mathbf{s}_t + \mathbf{n}_t, \quad t = 1, \ldots, T \tag{2}$$

where $\mathbf{y}_t = [y_1(t), ..., y_M(t)]^T$, $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)]$ is the array steering matrix, $\mathbf{s}_t = [s_1(t), \ldots, s_K(t)]^T$ contains the source signal amplitudes, $\mathbf{n}_t = [n_1(t), \ldots, n_M(t)]^T$ is the complex independent white Gaussian noise vector with zero mean and covariance $\sigma^2\mathbf{I}_M$. Here, T is the number of snapshots, and $y_m(t)$ and $n_m(t)$, $m = 1, \ldots, M$, are the output and measurement noise of the mth sensor at time t, respectively.

Collecting the T snapshots, the matrix form of (2) can be formulated as a multiple measurement vectors (MMV) model, given by

$$\mathbf{Y} = \mathbf{A}(\theta)\mathbf{S} + \mathbf{N} \tag{3}$$

where $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_T] \in \mathbb{C}^{M\times T}$, $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_T] \in \mathbb{C}^{K\times T}$ and $\mathbf{N} = [\mathbf{n}_1, \ldots, \mathbf{n}_T] \in \mathbb{C}^{M\times T}$.

In our study, we assume that K is known *a priori* and employ the $M \times K$ measurement matrix $\mathbf{Y_{sv}}$ by thresholding the K largest singular values of the $M \times T$ measurement matrix $\mathbf{Y}$ to reduce computational complexity in directly processing (3), which is analogous to the ℓ_1-SVD algorithm [4]. In summary, the problem of DOA estimation in sparse representation framework is to find the unknown DOAs given K, $\mathbf{Y_{sv}}$ and the mapping $\boldsymbol{\theta} \to \mathbf{A}(\boldsymbol{\theta})$.

2.2. DOA estimation in sparse representation framework

Let the set $\mathbf{\Phi} = \{\hat{\theta}_1, \ldots, \hat{\theta}_N\}$ be the discretized sampling grids of all potential directions in the admissible DOA range $[0, \pi]$, where N is the number of grid points and typically $N \gg M > K$. When the true DOAs are located at (or close to) the sampling grids, the typical DOA estimation model based on the sparse representation framework is linear:

$$\mathbf{Y_{sv}} = \mathbf{A}(\hat{\theta})\hat{\mathbf{S}} + \hat{\mathbf{N}} \tag{4}$$

where $\hat{\mathbf{S}} \in \mathbb{C}^{N\times K}$ is the sparse signal matrix and $\mathbf{A}(\hat{\theta}) = [\mathbf{a}(\hat{\theta}_1), \ldots, a(\hat{\theta}_N)] \in \mathbb{C}^{M\times N}$. The K rows in $\hat{\mathbf{S}}$ with largest magnitudes are identical to those of $\mathbf{S}$, and the remaining $N-K$ rows in $\hat{\mathbf{S}}$ are regarded as zero. In compressed sensing theory, the main task in (4) is to recover $\hat{\mathbf{S}}$ from the underdetermined system, and DOA estimation is equivalent to finding the positions of K nonzero rows in $\hat{\mathbf{S}}$. The sparse signal recovery can be formulated as the ℓ_0 norm constrained minimization problem:

$$(\ell_0): \quad \min_{\hat{\mathbf{S}}} \| \hat{\mathbf{S}} \|_{row,0} \quad s.t. \quad \mathbf{Y_{sv}} = \mathbf{A}(\hat{\theta})\hat{\mathbf{S}} + \hat{\mathbf{N}} \tag{5}$$

where $||\cdot||_{\text{row, 0}}$ counts the nonzero rows.

Since ℓ_0 norm function is highly discontinuous and nonconvex, solving the ℓ_0 norm constrained minimization problem is known to be NP-hard in general. To address this issue, the ℓ_1 norm, which is the closest convex norm to the ℓ_0 norm, is employed instead. Then the sparse signal recovery problem under the ℓ_1 norm function is:

$$(\ell_1): \quad \min_{\hat{\mathbf{S}}} \| \hat{\mathbf{s}}^{\ell_2} \|_1 \quad s.t. \quad \| \mathbf{Y_{sv}} - \mathbf{A}(\hat{\theta})\hat{\mathbf{S}} \|_F^2 \le \eta \tag{6}$$

where η is an upper-bound on the noise power, and $\hat{\mathbf{s}}^{\ell_2}$ is a function of $\hat{\mathbf{S}}$ whose the ith element equals the Frobenius norm of the ith row of $\hat{\mathbf{S}}$, i.e., $[\hat{\mathbf{s}}^{\ell_2}]_i = \| \hat{\mathbf{S}}(i,:) \|_2$. Numerical methods [4,18] have been presented for (6). However, larger coefficients are penalized more heavily in ℓ_1 norm than smaller coefficients, which results to that the sparsest solution of ℓ_1 norm penalty does not approximate the ℓ_0 norm penalty. Nevertheless, reweighted ℓ_1 norm minimization algorithms are designed to tackle this imbalance in (6):

$$(\mathrm{W}\ell_1): \quad \min_{\hat{\mathbf{S}}} \| \mathbf{W}(\hat{\mathbf{s}})^{\ell_2} \|_1 \quad s.t. \quad \| \mathbf{Y_{sv}} - \mathbf{A}(\hat{\theta})\hat{\mathbf{S}} \|_F^2 \le \eta \tag{7}$$

where $\mathbf{W}$ is a weighting matrix and has different forms according to different optimization criteria [19,20].

To this end, there are two main drawbacks of the DOA estimation methods based on ℓ_1 norm minimization: (i) they recover the DOAs only if the targets exactly correspond to the discretized sampling grids. However, the target positions are not precisely on the grids in practical scenarios and thus DOA estimation bias exists. Moreover, most conventional sparsity-based DOA estimation methods tackle this problem by using dense sampling grids, which lead to high computational complexity and the estimated DOAs are still constrained on the grids; (ii) they apply toolbox to calculate the ℓ_1 norm constrained minimization problem, such as CVX [21] and Sedumi [22], which cannot tackle the nonconvex optimization problem and is time-consuming, especially for large data size.