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A semi-analytical framework for structural reliability analysis

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Abstract

Estimation of the probability of failure of structural systems can often be computationally intensive and time consuming. Hence, alternative techniques for efficient computation of the probability of failure of complex structures while retaining the accuracy is of paramount importance to structural engineering community. This paper presents a semi-analytical approach for computing failure probability of structure. The proposed approach utilizes (i) polynomial correlated function expansion, a novel surrogate modelling technique, to determine the moments and (ii) maximum entropy method to determine an analytical expression for the response probability density function (PDF). Once the analytical expression for the PDF is obtained, failure probability is calculated by numerically integrating the PDF in the failure domain. As a special case, when only the first two moments of response are considered, an analytical formula for the probability of failure is also proposed. Four structural engineering problems are shown to illustrate the performance of the proposed approach. It is observed that the proposed approach outperforms other existing approaches.

Keywords: Maximum entropy method; Polynomial correlated function expansion; Probability of failure; Homotopy algorithm; Stochastic computation

1. Introduction

Uncertainties in geometric configurations, boundary conditions, material properties and applied loadings are inevitable in describing practical engineering systems. Various international standards [1–5] have accommodated this in an *ad hoc* way by introducing the use of safety factors at the design stage. However, such an approach is becoming less satisfactory in today's competitive design environment. Therefore, accurate evaluation of probability of failure of a structural system has become a fundamental problem.

Theoretically, evaluation of the probability of failure can be easily accomplished by solving a multivariate integral in the failure domain. However, the difficulty arises due to (a) irregularities in problem domain and/or (b) implicit nature of the function separating the safe and failure domains. As a consequence, standard integration rules are not directly applicable. The most popular method for determining the probability of failure is Monte Carlo simulation (MCS) [6–8]. The foundation of this method is based on an algorithm for generating random numbers. The probability of failure is obtained by carrying out simulations at large number of randomly generated sample points and

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counting the number of responses for which threshold limit is exceeded. Although MCS is straightforward, it can be highly time consuming when the system under consideration is complex. In order to reduce the computational effort, researchers have suggested various methods, such as first-order reliability method (FORM) [9], second-order reliability method (SORM) [10–12], response surface methods (RSM) [13–17], kriging based surrogate [18–20], high-order stochastic response surface method (HO-SRSM) [21], radial basis function (RBF) based surrogate [22–24], polynomial chaos expansion [25,26], high dimensional model representation [27–30], neural-network [31,32] *etc*. All these methods are computationally efficient but at the expense of accuracy.

A distinct approach, than those discussed above, is the statistical response characterization method [33]. This method expresses the response PDF by an explicit equation and utilizes numerical integration to obtain the probability of failure. Researchers have attempted to extend the spectral stochastic finite element method (SSFEM) [34], for probabilistic representation of response quantities. However, Sudret and Der Kiureghian [35] have demonstrated its limitation, specifically for problems involving small failure probabilities. An alternative way to obtain the PDF of response is to couple MCS and fitting techniques. In this approach, an assumed PDF is fitted to the obtained responses using MCS. The limitations of this method are mainly twofold: (a) determination of responses using MCS is computationally intensive and time consuming and (b) the accuracy of this approach is highly dependent on the assumed PDF.

A competent approach to determine the PDF of response is based on maximum entropy (ME) method. ME method was originally suggested by Jaynes [36], as an efficient way to predict PDF of response with limited knowledge. This approach yields the most unbiased distribution among all possible distributions. However, one limitation that accompany this method is the prior knowledge of moments.

The purpose of this paper is to present a new algorithm for computing the probability of failure of structure, with significantly enhanced efficiency. A key feature of the proposed approach resides in its capability to deal with both independent and dependent random variables without the requirement of any *ad hoc* transformation. This method couples the ME method with polynomial correlated function expansion (PCFE) [37]. While PCFE is used to determine the moments of response, ME is utilized to determine an analytical expression for the PDF of response. Once the analytical expression for the PDF is obtained, probability of failure is calculated by numerically integrating the PDF in the failure domain. As a special case, when only the first two moments of response are considered, an analytical formula for the probability of failure is also proposed.

Rest of the paper is organized as following. After setting up the problem in Section 2, a brief review of ME method and PCFE is provided in Section 3 and Section 4 respectively. The new algorithm, referred here as ME–PCFE, is presented in Section 5. The efficiency and accuracy of the proposed approach is illustrated with four examples in Section 6. Finally Section 7 presents the concluding remarks.

2. Problem setup

Suppose, $\mathbf{x} = (x_1, x_2, \ldots, x_N) : \Omega_{\mathbf{x}} \to \mathbb{R}^N$ be a N-dimensional random vector with cumulative distribution function $F_{\mathbf{x}}(\mathbf{X}) = P(\mathbf{x} \leqslant \mathbf{X})$, where P denotes probability, $\Omega_{\mathbf{x}}$ is the probability space and $\mathbf{x} \in \mathbb{R}^N$. Traditionally, a reliability problem is defined by a performance function, often known as limit-state function $\mathcal{J}(\mathbf{x})$, where $\mathcal{J}(\mathbf{x}) < 0$ denotes the failure domain $\Omega_{\mathbf{x}}^F$ and $\mathcal{J}(\mathbf{x}) \geqslant 0$ denotes the safe region, *i.e.*,

$$\Omega_{\mathbf{x}}^F \triangleq \{\mathbf{x} : \mathcal{J}(\mathbf{x}) < 0\}. \tag{1}$$

The failure probability P_f is defined as

$$P_f = P\left(\mathbf{x} \in \Omega_{\mathbf{x}}^F\right) = \int_{\Omega_{\mathbf{x}}^F} dF_{\mathbf{x}}(\mathbf{X}) = \int_{\Omega_{\mathbf{x}}} \xi_{\Omega_{\mathbf{x}}^F}(\mathbf{x})\, dF_{\mathbf{x}}(\mathbf{X}) \tag{2}$$

where $\xi_{\mathcal{C}}(\mathbf{x})$ is a characteristics function and satisfies

$$\xi_{\mathcal{C}}(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \mathcal{C} \\ 0 & \text{if } \mathbf{x} \notin \mathcal{C}. \end{cases} \tag{3}$$

It is apparent that the limit-state function $\mathcal{J}(\mathbf{x})$ plays an important role in determining the probability of failure. However, almost for all practical cases, explicit form for $\mathcal{J}(\mathbf{x})$ is not known and one need actual modelling and simulations to determine $\mathcal{J}(\mathbf{x})$.