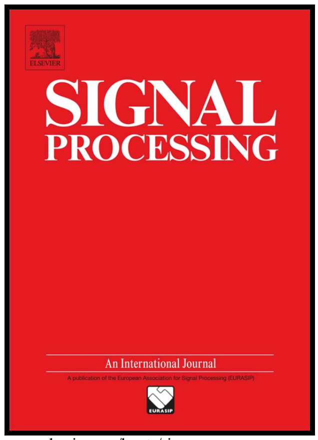
Author's Accepted Manuscript



www.elsevier.com/locate/sigpro

Variance Analysis of Unbiased Complex-valued ℓ_p-Norm Minimizer

Yuan Chen, Hing Cheung So, Ercan Engin Kuruoglu, Xiao Long Yang

PII: S0165-1684(16)30370-X
DOI: http://dx.doi.org/10.1016/j.sigpro.2016.12.018
Reference: SIGPRO6346

To appear in: *Signal Processing*

Received date: 20 October 2016
Revised date: 29 November 2016
Accepted date: 16 December 2016

Cite this article as: Yuan Chen, Hing Cheung So, Ercan Engin Kuruoglu anc Xiao Long Yang, Variance Analysis of Unbiased Complex-valued ℓ_p-Norr Minimizer, *Signal Processing,* http://dx.doi.org/10.1016/j.sigpro.2016.12.018

Variance Analysis of Unbiased Complex-valued ℓ_p-Norm Minimizer

Yuan Chen[1], Hing Cheung So[2], *EURASIP Member, Ercan Engin Kuruoglu*[3], *EURASIP Member, Xiao Long Yang*[1]

Abstract

Parameter estimation from noisy complex-valued measurements is a significant topic in various areas of science and engineering. In this aspect, an important goal is finding an unbiased estimator with minimum variance. Therefore, variance analysis of an estimator is desirable and of practical interest. In this paper, we concentrate on analyzing the complex-valued ℓ_p-norm minimizer with $p \geq 1$. Variance formulas for the resultant nonlinear estimators in the presence of three representative bivariate noise distributions, namely, α-stable, Student's t and mixture of generalized Gaussian models, are derived. To guarantee attaining the minimum variance for each noise process, optimum selection of p is studied, in the case of known noise statistics, such as probability density function and corresponding density parameters. All our results are confirmed by simulations and are compared with the Cramér-Rao lower bound.

Keywords: Variance analysis, complex-valued signals, fractional lower-order moment, ℓ_p-norm minimization, digamma function, Taylor series expansion

1. Introduction

In many areas of science and engineering, such as wireless communications, sensor array signal processing and biomedical sciences [1]–[3], observations are more conveniently modeled as complex-valued data, which have a simpler analytical form and is easier to deal with than the real-valued model. Parameter estimation for the complex-valued observations [4]–[5] is an important research topic and has attracted a great deal of attention. For numerous estimators developed in the literature, the goal is to find one which on the average yields the true value and the mean square error (MSE) between the estimate and true value is the smallest. MSE analysis of unbiased estimators is significant to help searching for one with minimum variance. Since the definition of variance relates to expectation, calculating by excessive simulations may be nonconclusive and unrealistic. To obtain the variance elegantly and correctly, approaches such as Taylor series expansion (TSE) on the estimates [6] and on the error function [7] are proposed, which are verified in [8]. However, they only consider estimators in the presence of complex Gaussian noise, which cannot describe other types of disturbances, especially those with impulsive nature, appeared in many fields [9]–[11]. For example, symmetric α-stable (SαS), Student's t and mixture of generalized Gaussian (MGG) processes are commonly employed to model the complex-valued impulsive noise.

[1]School of Computer & Communication Engineering, University of Science & Technology Beijing, Beijing, China
[2]Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, China
[3]Italian National Council of Research (ISTI-CNR), Pisa, Italy