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Projection Matrix Design Using Prior Information in Compressive Sensing

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Abstract

This paper proposes a projection matrix design algorithm using prior information on sparse signal to reduce local cumulative coherence, since small local coherence can improve the sparse signal recovery rate. Local cumulative coherence describes the coherence between the atoms indexed by the support of the sparse signal and other atoms. Using prior information on the sparse signal, projection matrix design is formulated as an optimization problem that minimizes the weighted Frobenius distance between the Gram matrix and the identity matrix. This optimization problem is solved by majorization-minimization method, which iteratively minimizes the surrogate function. If the prior information is accurate, the designed projection matrix can make the local cumulative coherence small. Numerical experiments on both synthesized signals and real image sequences demonstrate the effectiveness of the proposed algorithm in improving the performance of sparse signal recovery algorithms such as greedy algorithms and basis pursuit algorithm.

Keywords: Compressive sensing, sparse signal recovery, projection matrix, prior information, coherence.

1. Introduction

Compressive sensing aims to recover a high dimensional sparse signal from its low dimensional linear projection measurements [1]. Compressive sensing has been successfully applied to channel estimation [2], direction of arrival (DOA) estimation[3] and image denoising[4]. In compressive sensing, let $\boldsymbol{\alpha}$ be an l dimensional real signal. Its insufficient linear measurement can be written as

$$\boldsymbol{y} = \boldsymbol{P}\boldsymbol{\alpha} + \boldsymbol{e} \quad (1)$$

where $\boldsymbol{P} \in \boldsymbol{R}^{m\times l}$ is the projection matrix, $\boldsymbol{y} \in \boldsymbol{R}^{m\times 1}$ is the measurement signal and $\boldsymbol{e} \in \boldsymbol{R}^{m\times 1}$ is the measurement noise. Here, insufficient measurement means that the length of measurement signal $\boldsymbol{y}$, which is m, is less than that of signal $\boldsymbol{\alpha}$. This makes the signal recovery problem an indefinite linear equation, which has an infinite number of solutions.

The crucial assumption in compressive sensing is that the signal $\boldsymbol{\alpha}$ can be sparsely represented by a redundant or non-redundant basis, which can be written as

$$\boldsymbol{\alpha} = \boldsymbol{D}\boldsymbol{x} \quad (2)$$

where $\boldsymbol{D} \in \boldsymbol{R}^{l\times n}$ is called the sparsifying dictionary and $\boldsymbol{x} \in \boldsymbol{R}^{n\times 1}$ is a sparse representation of $\boldsymbol{\alpha}$. With (2) we denote that signal $\boldsymbol{\alpha}$ can be sparsely represented by matrix $\boldsymbol{D}$ and the representation coefficient $\boldsymbol{x}$ is a sparse signal [5]. Here, sparse signal means that the number of its nonzero components is far fewer than the length of the signal itself [1].

The assumption that signal $\boldsymbol{\alpha}$ can be sparsely represented is reasonable, since many signals that we deal with are sparse themselves or are sparse in some domains [6][7].

With (1) and (2), the measurement signal can be written as

$$\boldsymbol{y} = \boldsymbol{P}\boldsymbol{D}\boldsymbol{x} + \boldsymbol{e} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{e} \quad (3)$$

where $\boldsymbol{\Phi} = \boldsymbol{P}\boldsymbol{D}$. $\boldsymbol{\Phi}$ is termed the equivalent measurement matrix. In the following, we abbreviate the equivalent measurement matrix as measurement dictionary.

With the assumption that $\boldsymbol{x}$ is sparse, the signal recovery can be formulated as the following optimization problem:

$$\min_{\boldsymbol{x}\in\boldsymbol{R}^{n\times 1}} \|\boldsymbol{x}\|_0 \quad s.t.\ \|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{x}\|_2 \le \sigma \quad (4)$$

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