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# A joint framework for multivariate signal denoising using multivariate empirical mode decomposition

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ABSTRACT

In this paper, a novel multivariate denoising scheme using multivariate empirical mode decomposition (MEMD) is proposed. Unlike previous EMD-based denoising methods, the proposed scheme can align common frequency modes across multiple channels of a multivariate data, thus, facilitating direct multichannel data denoising. The key idea in this work is to extend our earlier MEMD based denoising method for univariate signal in Hao et al. (2016) [19] to the multivariate data. The MEMD modes (known as intrinsic mode functions) for separating noise components are first adaptively selected on the basis of a similarity measure between the probability density function (pdf) of the input multivariate signal and that of each mode by Frobenius norm. The selected modes are then denoised further by a local interval thesholding procedure followed by reconstruction of the thresholded IMFs. The resulting method operates directly in multidimensional space where input signal resides, owing to MEMD, and also benefits from its mode-alignment property. Furthermore, subspace projection is introduced within the framework of the proposed method to exploit the inter-channel dependence among IMFs with the same index, enabling the diversity reception of the signal. Performance of the proposed method against standard multiscale denoising schemes is demonstrated on both synthetic and real world data.

## 1. Introduction

Real world data is often corrupted with unwanted noise which must be removed before further signal processing. Existing denoising algorithms, such as the least mean square (LMS) based Wiener and Kalman filtering [1], multi-scale analysis based wavelet denoising [2] and the newly developed empirical mode decomposition (EMD) method [3], are mainly designed for univariate signals. However, with the development of multichannel sensor technology, multivariate denoising is urgently needed in many applications ranging from communication system [4] to biomedicine [5].

Due to the fact that most of the signals are sparse in wavelet domain, thresholding or shrinking based nonlinear operators are employed in wavelet denoising [6] and it has been confirmed to be very effective for practical 1D and 2D (image) signals. In order to denoise multichannel signals jointly, a multivariate extension of univariate wavelet denoising (MWD) method was presented in [7]. With a combination of $p$ univariate wavelet denoising and principal component analysis (PCA) performed on the signal after decorrelating the noise among channels, MWD method achieves better performance than traditional channel-wise wavelet denoising. Recently, synchrosqueezing transform (SST) has been introduced to the wavelet denoising and it has been confirmed that it can outperform a state-of-the-art methods based on wavelets for the signals with weak frequency modulation [8]. Consequently, multivariate wavelet synchrosqueezing denoising (MWSD) was proposed in [9] which employs the thresholding technique for the multivariate oscillatory framework.

EMD is a fully data-driven time-frequency analysis method, which has been widely applied to analyze nonlinear and nonstationary signals [10]. Unlike standard approaches such as Fourier and wavelet transform that project input data onto predefined and fixed basis functions, EMD decomposes an arbitrary signal into a complete and finite set of localized amplitude/frequency modulated (AM/FM) oscillations derived from the signal itself, which are called intrinsic mode functions (IMFs). Based on the above properties, EMD based denoising methods have been shown to outperform the wavelet based ones [11].

Specifically, for the purpose of signal denoising, the following EMD based approaches have emerged recently: direct thresholding EMD (EMD-DT) methods which have been inspired by the wavelet thesholding proposed by Donoho [6]. In this method, the filtered signal is obtained by thresholding the IMFs directly before signal reconstruction [12]. To be consistent with the characteristics of the IMF, interval thresholding was presented in [11] and was shown to outperform EMD-DT. In [13], partial reconstruction of IMFs was used to perform

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signal denoising. However, it also raises the question on how to select relevant modes (IMFs) within the framework of EMD. When EMD is applied to a noisy signal, it is necessary to determine whether the resulting IMF belongs to noise or signal. Correlation-based thresholding was adopted to choose the relevant IMFs in [14,15], respectively. But for a noisy signal with different signal-to-noise ratio (SNR), both these methods have been found to be unstable, owing to strong or weak correlation between the noisy signal and the first IMF. To avoid this shortcoming, a more robust method using similarity measure between the probability density function (pdf) of the input signal and that of each IMF was presented in [16].

However, the problem of uniqueness within univariate EMD still leads to a serious obstacle: there is no guarantee that the same-index IMFs from multiple channels carry the information pertaining to the same scale to facilitate the analysis of multichannel signals [17]. Moreover, standard EMD may suffer from mode-mixing and spectral aliasing for intermittent data [18]. Mode-mixing, whereby either an IMF contains different oscillatory modes or one mode is appeared in different IMFs, makes it hard to obtain a clean signal via partial reconstruction. The performance of thresholding methods depends on an accurate estimate of noise energy, and thus they are also limited by spectral aliasing.

To resolve this issue, we have proposed to use a recent multivariate extension of EMD, namely multivariate empirical mode decomposition (MEMD), to analyze univariate data and thus circumvent the problem of uniqueness in [19]. However, the denoising method in [19] is limited to the univariate signal under Gaussian noise. In this paper, we propose a novel MEMD based denoising method for multichannel data. The method is a nontrivial extension of [19] for multivariate data owing to the fact that we employ subspace projection scheme in conjunction with MEMD for improved denoising performance. Furthermore, a new mechanism for the estimation of the pdfs of multivariate data has been employed in this work, which is fundamentally different from the univariate pdf estimation used in [19]. The proposed scheme operates by decomposing a multivariate data via noise-assisted MEMD [20], followed by an estimation of the pdf of each extracted joint rotational mode. Interval thresholding and partial reconstruction are next applied to denoise the IMFs based on the value of a similarity measure between the pdf of the input signal and that of each mode via Frobenius norm. Moreover, subspace projection is adopted to denoise multivariate signals further by utilizing the channel diversity. To illustrate the effectiveness of the proposed scheme, simulations on both synthetic and real world signals are conducted to support the analysis.

The organization of this paper is as follows. Section 2 provides a brief description of MEMD. Section 3 addresses the MEMD based multivariate denoising and Section 4 describes MEMD combined with subspace projection based multivariate denoising. Section 5 validate the performance of the proposed algorithm through simulations, and the final conclusions are drawn in Section 6.

## 2. Multivariate empirical mode decomposition

Standard univariate EMD is only applicable to univariate or single-channel input data and divides it into a combination of oscillatory modes. MEMD has been recently developed to process a general class of multivariate signals having an arbitrary number of channels. MEMD extends the notion of extracting "oscillations" in univariate EMD to extracting "rotations" in multidimensional space [21].

For an $n$-variate signal (containing $n$ number of channels) the local mean cannot be defined directly, and thus MEMD takes multiple projections of the input signal along uniformly sampled directions based on a low discrepancy Hammersley sequence. Once the projections in multidimensional space are obtained, the local mean of the multivariate signal can be calculated by averaging all the envelopes along the sampled directions in ($n$-1)-dimensional space. To utilize the benefits of quasi-dyadic filter bank structure of MEMD on white Gaussian noise (WGN), $l$-channel independent white noise is added to $n$-channel multivariate data to create an ($n$+$l$)-dimensional "composite" space, and then process such a composite signal via MEMD, namely noise-assisted MEMD method [22]. Since the added noise channels have a broad range in the frequency spectrum, MEMD aligns the IMFs corresponding to the input signals according to the structure of the dyadic filter bank, which also helps to reduce mode-mixing within the extracted IMFs. Algorithm 1 lists the steps involved in the MEMD algorithm.

Fractional Gaussian noise (fGn) is a general case of white Gaussian noise and is found to be very efficient to describe the long-range dependence of noise via Hurst exponent $H$. For the special case $H$=0.5, fGn reduces to the white Gaussian noise. Both EMD and MEMD have exhibited excellent filter bank properties for fGn in statistic in [23,24]. However, we are more concerned about the decomposition result for only a single realization from the perspective of practical application. As the dyadic filter bank property of MEMD for fGn is not sensitive to the Hurst of the assisted fGn [24], thus for the univariate fGn process of length $N$=1000, MEMD is applied with two WGN channels and EMD is applied directly. The frequency response and the corresponding filter bank property are illustrated based on a single realization of an univariate fGn in Fig. 1. It can be observed that the IMFs obtained from MEMD exhibit a better quasi-dyadic filter bank structure than EMD, irrespective of Hurst exponent $H$. The alignment of IMFs with MEMD results in more stable individual spectra and thus leads to a better estimation of noise-only IMF energy, which is very important for the thresholding process.

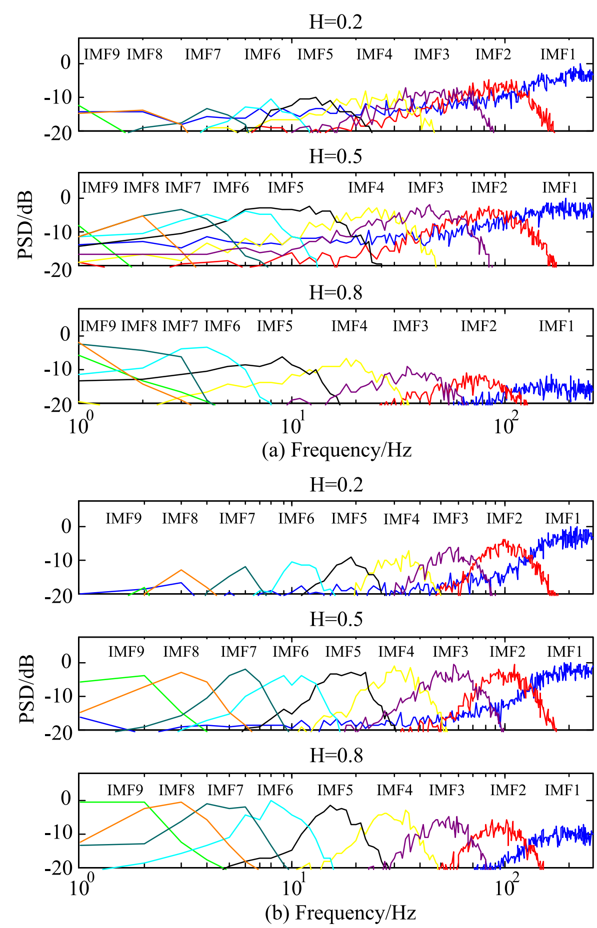


**Fig. 1.** Spectra of IMF1-IMF9 obtained from a single realization of an univariate fGn decomposed via (a) EMD and (b) MEMD.