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A semi-parametric polynomial frequency response estimator for identification of weakly non-linear systems using multi-amplitude MLS measurements - part II: Extension and application of the H_p estimator☆

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ABSTRACT

In this article the family of polynomial estimators for the estimation of the linear system response of weakly non-linear systems is examined regarding estimation errors and applicability. The H_{p3} estimator is introduced as an extension for measurement setups having both input and output noise and it is shown how the results obtained with the polynomial estimators are related to the parameters of Volterra models. The bias-variance tradeoff of the estimation error is derived and methods for obtaining the minimum error are presented.

1. Introduction

Weakly non-linear systems are systems which are intended to be linear but suffer from unwanted non-linear distortions which are present in most real, non-ideal physical systems. The estimation of the frequency response function (FRF) of such systems can either be done by purely linear identification approaches which only lead to a best linear approximation (BLA) [1,2] or by the complete parametric identification of more or less complex non-linear system models as e.g. the parallel-cascade model or special cases of it as e.g. in [3–6], or even the more general Volterra series [7]. The linearization on one hand of course will suffer from a prediction error for the output signal, whose magnitude depends on the amount of non-linearity in the system under test (SUT) and the level of the input signal. The Volterra approaches on the other hand are very complex, hence their application is limited to systems with short memory and low orders of non-linearity, as e.g. in [8–10].

In the preceding article [11], we introduced the polynomial H_p estimators as an extension of the well-known purely linear estimators H_1 and H_2 for the application to weakly non-linear systems. Two estimators for either output or input noise only, the H_{p1} and the H_{p2} estimator respectively were derived, based on a semi-parametric system model to overcome the limitations of linear approaches. We showed that they are consistent estimators for the true linear processing of weakly non-linear systems modeled by a Wiener-Hammerstein-type parallel-cascade model for these special measurement setups.

In the present article, we will consider measurement setups where input and output noise are present simultaneously. For linear systems there exist FRF estimators which are consistent in this case, see e.g. [12,13], called H_3 estimator. This method needs the test signal to be completely deterministic, which is not an issue when generated digitally, but allows for any linear pre-processing in order to excite the actual SUT, such as e.g. an amplification in order to drive a loudspeaker. We will adopt this method and extend the polynomial estimator family analogously by the new member H_{p3} for the estimation of the linear subsystem of weakly non-linear systems in the present of input and output noise. This will be done for linear pre-processing and then be further extended to non-linear pre-processing.

Besides this, we will show that these estimators can also be applied to systems represented by a finite Volterra series, hence showing consistency for the estimation of the linear FRF of an even wider class of weakly non-linear systems. This is done based on a brief analysis of the frequency domain representation of the Volterra series. The consistent estimation of the linear kernel is shown to be possible without the need of identifying the higher order kernels using the proposed estimator. Consequently, the complexity of our H_p estimator can be located in between the linearization and the non-linear model approaches. It will deliver better results than the BLA and is furthermore able to identify a synopsis of the Volterra parameters and in contrast to the complete identification is applicable to systems with long memory and high orders of non-linearity.

System identification processes are usually affected by measurement noise, which leads to variance errors and by bias errors that arise from systematic flaws, such as the wrong choice of the approximation

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order. The H_p estimator will be analyzed regarding its sensitivity to both types of errors. This will include the dependencies on noise variance, chosen approximation order as well as the actual system order. These considerations will then be used to develop methods to obtain a minimum overall error in practical applications.

This article is structured as follows: Section 2 will give a brief review of the H_p estimator family. We will then extend the family by a new member, the H_{p3} estimator. Section 3 will show that the H_p estimators are valid for all weakly non-linear systems for which a converging Volterra series exists. To further increase the practical applicability of the H_p estimators, the computation of the estimates will be modified within Section 4 in order to reduce the arithmetic load of the procedure by a simplified calculation of the pseudoinverse. In Section 5 we will analyze the H_p estimator family regarding error sources in detail and discuss ways to reduce the errors. These considerations will lead us to the so-called bias-variance tradeoff, which will be the basis for the development and discussion of methods for determining the in some sense best approximation order. These methods are presented and analyzed within simulations in Section 6, before finally Section 7 will close with a conclusion as well as some remarks on further research.

2. The H_{p3} estimator for systems with input and output noise

2.1. Short review of the polynomial H_p estimator

In [11] we developed a consistent frequency-response estimator for the linear processing of a non-linear SUT, where the derivation was done in the frequency domain as it is common practice for H_1 and related estimators. For this purpose we modeled the SUT with a Wiener-Hammerstein-type system shown in Fig. 1. We have shown, that for a given excitation signal, the system output Y_i at any frequency f_i can be expressed by a polynomial of order D with excitation signal X_i at the same frequency f_i as the variable and the so called equivalent path-filters (EPF) $\widetilde{G}_{k,i}$ as the coefficients

$$Y_i = \widetilde{G}_{1,i}\, X_i + \widetilde{G}_{2,i}\, X_i^2 + \cdots + \widetilde{G}_{D,i}\, X_i^D. \tag{1}$$

The system described by (1) can be interpreted as a simplified model with a Hammerstein-type path for each order containing the EPFs $\widetilde{G}_{k,i}$ as it is shown in Fig. 6. An EPF summarizes the contributions of the filters $H_{W,i}$ and $H_{H,i}$ at frequency f_i and because of the non-linearities in the paths it is signal dependent. Thus, this simplification is coupled with the restriction that the EPFs except that for the linear path are only valid for the used test signal. We propose pseudo-random, binary maximum length sequences (MLS) as test signals, which show constant magnitude response and are preferable test signals because of their low crest-factor in the time-domain. However in principle it is possible to use any deterministic signal as a test signal in conjunction with the proposed estimators, as long as it is sampled coherently.

In order to identify the EPFs we apply the test signal with N_A different amplitudes. By doing so we gain a system of N_A linear equations following (1) for each frequency f_i. This can be represented by a matrix multiplication

$$\mathbf{Y_i} = \mathbf{X_i}\, \mathbf{G_i}, \tag{2}$$

where $\mathbf{G_i}$ and $\mathbf{Y_i}$ are column vectors consisting of all D path weights and the output for all N_A measured amplitudes respectively, and $\mathbf{X_i}$ is a $N_A \times D$ Vandermonde-like matrix consisting of the N_A input amplitudes and its D exponents, each at frequency f_i. The number of measurements at different amplitudes is assumed to be higher than the system order, hence $N_A > D$. This makes $\mathbf{X_i}$ a non-squared matrix. In order to identify the EPFs $\mathbf{G_i}$ we therefore use the Moore-Penrose pseudoinverse of $\mathbf{X_i}$ as

$$\widehat{\mathbf{G}}_\mathbf{i} = (\mathbf{X_i^H}\, \mathbf{X_i})^{-1}\, \mathbf{X_i^H}\, \mathbf{Y_i}. \tag{3}$$

In general, a measurement is disturbed by stochastic noise. In our case, the system input X and the system output Y are considered to be disturbed by uncorrelated additive white Gaussian noise M_{in} and M_{out} respectively, so that the true input and output signals can only be estimated as $\widehat{X}$ and $\widehat{Y}$. For the noise contaminated measurement setup as shown in Fig. 2, two special cases were analyzed: for output noise only ($M_{in} = 0$) the estimator in (3) was shown to be consistent and therefore the non-linear extension of the well known H_1 estimator, hence it was called the H_{p1} estimator. Analogously for measurement setups with input noise only ($M_{out} = 0$) the non-linear extension of the consistent H_2 estimator

$$\widehat{\mathbf{G}}_\mathbf{i} = (\mathbf{Y_i^H}\, \widehat{\mathbf{X}}_\mathbf{i})^{-1}\, \mathbf{Y_i^H}\, \mathbf{Y_i} \tag{4}$$

was introduced as the H_{p2} estimator and its consistency was shown.

2.2. The H_{p3} estimator

Both polynomial estimators introduced by now are only consistent for special cases of measurement noise. This is output noise only in the case of the H_{p1} estimator and input noise only in the case of the H_{p2} estimator. A single noise source or more realistically, a single noise source that dominates is often a valid assumption, as e.g. the input noise can be neglected compared to the input signal. However, for some measurement setups none of these assumptions can be made and both types of noise sources need to be considered. In the following, we will discuss several non-linear system measurement setups with in- and output noise and develop special, consistent polynomial estimators for these cases.

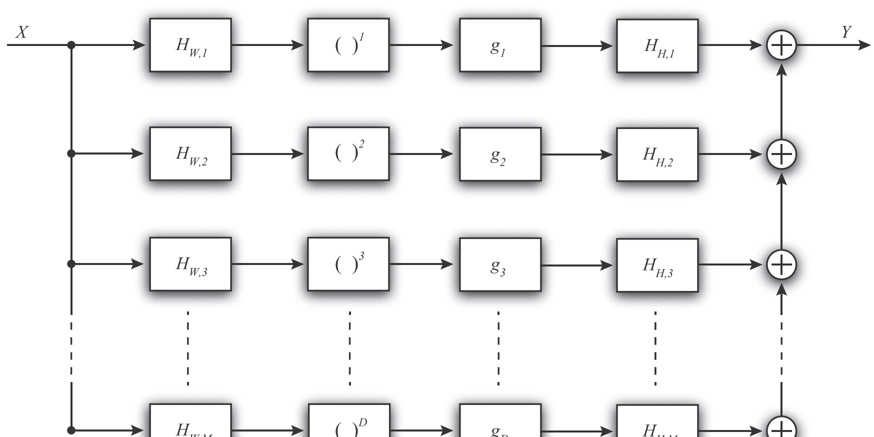

Fig. 1. Block-diagram of the Wiener-Hammerstein-type model.