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A semi-parametric polynomial frequency response estimator for identification of weakly non-linear systems using multi-amplitude MLS measurements☆

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ABSTRACT

In this article a method for the measurement of the linear frequency response of weakly non-linear systems is described. A frequency domain estimator is developed by analyzing the output spectra for single- and multitone excitations of several types of system models with the most complex system one being a parallel-cascade Wiener-Hammerstein-type model which is able to represent a wide range of weakly non-linear systems. It is then shown that the output spectrum of such a weakly non-linear system can be expressed by a polynomial with constant coefficients for a given input signal at each frequency. This leads to the proposed polynomial estimator H_p that is capable of identifying the true linear, nonparametric frequency response of the system under test. Special versions for either input or output noise only are developed and consistency is shown for all considered system models.

1. Introduction

Signal processing systems, whether analog or digital, are generally not perfectly linear, but usually include some kind of unwanted non-linear signal processing. This can be due to the physical behavior of circuit elements in analog systems or due to quantization in digital systems. We will refer to the class of systems that are intended to be linear but have some, in general not limited, amount of non-linear processing due to nonidealities as weakly non-linear systems.

In many cases the true linear processing rather than a linearization of the weakly non-linear system is of interest. For example, for audio systems the question, of which nature the non-linear processing artifacts are and if they are audible, is essential and longstanding in the field of audio quality assessment. It becomes possible to evaluate the effects of the non-linear processing on the output signal of a system under test SUT, if the components generated by the linear and non-linear processing in the output signal can be separated. This can be achieved by identification of the linear processing in the SUT, which is defined by the frequency response function (FRF) H_{lin}. With the knowledge of this function, we have a linear substitute system with which the linearly processed output and the non-linear distortions can be separated and analyzed individually. Consequently, the challenge is to identify this linear subsystem. A vast range of approaches for the task of the identification of linear system behavior have been proposed involving different system models for the SUT and different types of test signals.

There are various approaches to modeling and identification of weakly non-linear systems [1]. The most general approach to model such systems with finite memory is the use of Volterra series [2]. They have been discussed and applied in various publications, see e.g. [3–7]. In practice the estimation of the Volterra system parameters is limited to systems with short memories and low orders of non-linearity because the complexity of the model parameter identification grows significantly with increasing order. Additionally, the interpretation of the obtained parameters, especially in the time domain is not very intuitive.

Due to the relatively high complexity of a Volterra system, which is in many cases not needed for a certain application, simpler system models are used widely. The two basic models are the Hammerstein model, where a static non-linearity is followed by a linear time-invariant system, and the Wiener model, where the arrangement of the linear and the non-linear block is reversed. The static non-linearity is usually described by a characteristic curve, which can be approximated by a polynomial. Numerous publications [8–11] make use of one of these models to consider non-linear effects as a step of improvement over a purely linear system model.

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A wider range of system behavior can be described using a combination of these two simple models which leads to the Wiener-Hammerstein model. This system model consists of three blocks in series, a linear system followed by a static non-linearity followed by a second linear system. Research and applications of this model can be found in control theory, see e.g. [12–16]. By combining multiple parallel paths, each consisting of a Wiener-Hammerstein system, an even more general parallel-cascade model [17] is established. Most of the work in these models aims at determining a linear approximation for a given operating point, the so called best linear approximation (BLA) [18]. In [19] a least-squares estimation in the time-domain is used with multi-amplitude sweeps as test signals to obtain the linear response of a parallel-cascade model.

In this paper, we derive the proposed FRF estimator for each of the system models, from the Hammerstein model up to a special case of the parallel-cascade model, where we consider a monomial (instead of a complete polynomial) in each path with unique exponents in the range $1\ldots D$ to model a system of the given order D. We will refer to this model as a Wiener-Hammerstein-type model. In this model, each path has its own set of filters before and after the static non-linearity and additionally weighting factors g_k are introduced in each path. This model therefore allows to incorporate frequency dependent non-linear distortions through the path-filters. We consider these filters to be (non-parametric) FIR-filters with a length that is directly related to the analysis (test signal) length. The non-linearity is in contrast modeled parametrically, with the parameters being the g_k weights in each path. Hence our complete model is of semi-parametric nature. The model is non-recursive, but it is also appropriate to model recursive systems if we choose the length of the filters high enough.

Besides the system model, the choice of appropriate test signals and procedures is very important for the efficiency and the accuracy of the measurement and the system identification. Suitable test signals could be single tones, linear or exponential sweeps [20], multitones [21,22], gaussian white noise or pseudorandom signals like maximum length sequences (MLS) [23,24]. Two requirements on an efficient test signal are a low crestfactor to achieve a good signal to noise ratio (SNR) and periodicity to avoid the use of windowing. With narrowband signals like sines or sweeps, it is relatively simple to separate linear and non-linear processed components in the output signal of the SUT as e.g. in [25]. If the test signals are broad-band signals like multitones, the measurement can deliver information at multiple frequencies simultaneously and thus measurement speed can be increased compared to multiple single-tone measurements.

In the scope of this paper, we will use MLS because of their low crest-factor, their periodicity and therefore coherent sampling and their constant magnitude spectrum. MLS can be interpreted as full-band multitones with a minimum crest-factor in the time domain. Test signals with lower crest-factors can also be used with the proposed estimator, resulting in a lower SNR, hence more averaging is needed for the same quality of estimation. An iterative measurement using sets of maximum length and other pseudorandom binary sequences of equal length but with different phases has been applied in [26,27]. This method is able to reduce artifacts caused by non-linear distortion by phase randomization and averaging.

However, single-amplitude MLS measurements like these and also e.g. [13,15] can only determine the best linear approximation for a single excitation level of the system under test. One could visualize this as a linearization between two points on a characteristic curve where the measured gain is then the slope of the line defined by the two points. This gain is called equivalent gain, because it does not equal the gain of the linear part of the SUT, but is influenced by all odd-order distortions which add an amplitude dependent contribution to the true linear gain. Because the excitation of the system with a single-amplitude MLS can not reveal the true linear gain of the SUT, the measurement technique is extended by the use of a multi-amplitude test signal and hence an MLS is applied with multiple different amplitudes to identify the real (amplitude-independent) linear part of the SUT. With the proposed method this can be done without the need of identifying the non-linear processing of the system.

The paper is structured as follows. In the second section we will analyze the output spectra for each of the considered system models, starting from a system with a single non-linear block and no memory. These output spectra will be computed for single tone excitations and we will analyze how the non-linear distortion products influence the output of the linear subsystem at the excitation frequency. In the third section we will continue with the derivation of the output of the most general model when the system is excited by multitones, to show that the proposed H_p estimator can be applied with broadband test signals. This analysis gives a very detailed insight into the output of weakly non-linear systems, combining several analysis approaches in order to get an in-depth understanding of the spectral effects of non-linear signal processing. Section four finally introduces two variants of the proposed estimator that can be used to identify the linear part of the Wiener-Hammerstein-type system. Section five concludes the paper. Extensions, error analysis as well as practical considerations of the derived estimator will be given in a follow-up article [28].

2. Model of the system under test using single tone excitations

In the scope of this paper, we will consider all systems to have a smooth non-linearity which can be described by a differentiable (potentially frequency dependent) characteristic curve. Hence we will assume that all considered SUTs can be modeled by a parallel-cascade model with monomials as the non-linearity in each path. Thereby the highest monomial, which is equivalent to the system order D, can be arbitrary high but has to be finite or at least the SUT has to be approximable within the required accuracy by a truncated series of order D. In order to develop a consistent estimator we assume the system order D to be known a priori and the determination of D is not topic of this article.

We will now derive the formulas for computing the output of the Wiener-Hammerstein-type system models. The description will start with memoryless systems to motivate the concept of the amplitude response curve, evolve to Wiener and Hammerstein systems and finally the output spectrum of Wiener-Hammerstein-type parallel-cascade systems will be examined.

2.1. Memoryless static non-linearity

Nonlinear electric systems are commonly specified by their characteristic curve which determines their input-output dependency. This curve can be described, at least within a specified input region, by a simple polynomial of order D as

$$y(n) = \sum_{k=1}^{D} g_k\, x(n)^k \tag{1}$$

with the time-discrete, instantaneous values of the output $y(n)$, the input $x(n)$ and the weights g_k for each order k.

This kind of characteristic curves is well known from data sheets and they can be easily interpreted concerning constant (DC) input and output quantities. The output quantity is determined directly by the polynomial in (1). For alternating quantities the time dependency needs to be considered which complicates the interpretation for sinusoidal signals, where we are not interested in the instantaneous values but rather in the input- and output amplitudes. Since we are looking at the frequency domain representation of a system, we will explore the influence of such weak non-linearities on sinusoidal input signals by means of the input and output spectral amplitudes.