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A parallel matrix-free conservative solution interpolation on unstructured tetrahedral meshes

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Abstract

This document presents an interpolation operator on unstructured tetrahedral meshes that satisfies the properties of mass conservation, $\mathbb{P}_1$-exactness (order 2) and maximum principle. Interpolation operators are important for many applications in scientific computing. For instance, in the context of anisotropic mesh adaptation for time-dependent problems, the interpolation stage becomes crucial as the error due to solution transfer accumulates throughout the simulation. This error can eventually spoil the overall solution accuracy. When dealing with conservation laws in CFD, solution accuracy requires enforcement of mass preservation throughout the computation, in particular in long time scale computations. In the proposed approach, the conservation property is achieved by local mesh intersection and quadrature formulae. Derivatives reconstruction is used to obtain a second order method. Algorithmically, our goal is to design a method which is robust and efficient. The robustness is mandatory to obtain a reliable method on real-life applications and to apply the operator to highly anisotropic meshes. The efficiency is achieved by designing a matrix-free operator which is highly parallel. A multi-thread parallelization is given in this work. Several numerical examples are presented to illustrate the efficiency of the proposed approach.

Keywords: Solution interpolation; Matrix-free conservative interpolation; Parallel interpolation; Unstructured mesh; Mesh adaptation; Conservation laws

1. Introduction

Solution interpolation or solution transfer is an important stage for several applications in scientific computing. For instance, this stage is important for coupled problems, e.g. multi-physics simulations or fluid–structure interaction (FSI) problems, where specific meshes are considered for each sub-problem, see [1,2]. The preservation of the conservation property during the interpolation stage is also crucial for the accuracy in long time scale simulations, see [3,4]. It is an essential component of Arbitrary Lagrangian–Eulerian (ALE) methods as well. An accurate remapping must satisfy several properties such as conservation, high order accuracy, bound preserving, etc. This is also a key point in the context of time-accurate mesh adaptation. Indeed, it links the mesh generation and the

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numerical flow solver allowing the simulation to be restarted from the previous state. More precisely, after generating a new adapted mesh, called *current mesh*, the aim is to recover the solution field defined on the previous mesh, called *background mesh*, on this new mesh to pursue the computation. This recurrent stage in adaptive simulations is crucial for time-accurate unsteady problems as errors introduced by the interpolation procedure accumulate throughout the computations. The negative impact of such errors on solution accuracy was pointed out in [5] where standard linear interpolation is applied. In [6], it has been demonstrated – for the 2D case – the importance of a conservative interpolation operator on the accuracy of anisotropic time-accurate mesh adaptation for conservation laws.

A conservative interpolation based on a Galerkin projection has been proposed in [1,7,8]. The Galerkin projection requires the assembling and the resolution of a linear system. This method can as well suffer from oscillations. To bound the projected quantities specific treatments have to be done [7]. A global supermesh construction is used in [7,8] but it limits considerably the efficiency of the method and the maximal size of the problem in 3D due to memory usage. A local approach has been proposed in [6] which improves the algorithm efficiency. It is based on a local mesh intersection procedure and local quadrature. This local approach has been followed in [9] coupled with local Galerkin projection. In [6], the maximum principle is strictly enforced using L^2-optimal gradient correction, the method is thus free from oscillations.

In this paper, we consider the 3D solution interpolation for anisotropic adapted tetrahedral meshes where the background and the current meshes are distinct, in the sense that the number of entities and the connectivities can be completely different. Flows are modeled by the conservative compressible Euler equations and resolved by a second order finite volume scheme. Therefore, to obtain a consistent mesh adaptation loop, the proposed interpolation scheme must satisfy the following properties:

- mass conservation
- $\mathbb{P}_1$ exactness implying an order 2 for the method
- maximum principle.

Moreover, this method has to be algorithmically very robust as we deal with highly stretched elements and it has to be very efficient to be applied to real-life applications. The word efficient signifies that it requires low memory storage and that the additional required CPU time over that for standard linear interpolation is acceptable. In consequence, we propose a *matrix-free* approach based on local mesh intersections and appropriate local reconstructions.

The mass conservation property of the interpolation operator is achieved by local mesh intersection, *i.e.,* intersections are performed at the element level. The use of mesh intersection for conservative interpolation seems natural for unconnected meshes and has already been alluded in [10] or applied in [11] for order 1 reconstruction. The locality is inherent for efficiency and robustness. Once again for efficiency purposes, the proposed intersection algorithm is especially designed for simplicial meshes. The idea is to compute the intersection between two simplexes, mesh this intersection, and use a quadrature formulae to exactly compute the transferred mass. Moreover, the designed algorithm is highly scalable in parallel due to its locality.

The high-order accuracy is obtained by a solution gradient reconstruction from the discrete data and the use of Taylor formulae. This high-order interpolation can lead to loss of monotonicity. The maximum principle is then enforced by correcting the interpolated solution, thus the interpolated solution is free from any oscillations. Notice that much care has been taken while designing the localization algorithm as it is also critical for efficiency.

The proposed $\mathbb{P}_1$-conservative interpolation operator is suitable for solutions defined at elements or vertices.

This paper is the extension of [6] to the three-dimensional case. As compared to [6], several novelties are presented in this work. They are required to ensure the robustness,[1] efficiency and accuracy of the method when dealing with 3D highly anisotropic meshes on complex geometries. These novelties concern:

- A specific tetra–tetra intersection procedure where floating point arithmetic is treated with a particular care, as it must be extremely robust and able to deal with highly anisotropic tetrahedra (anisotropic ratio up to 10^5).
- A new method to mesh the tetra–tetra intersection ensuring extra numerical accuracy.
- Solutions to deal with the case of non-matching boundaries between meshes (this is always the case for complex geometries), this is crucial to not introduce artifacts in the solution during the interpolation stage and preserve accuracy.

[1] Ensuring the robustness of the local mesh intersection and the meshing of these intersections is considerably more difficult in 3D than in 2D. Indeed, volume positivity is harder to satisfy than area positivity when dealing with degenerated cases.