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Robust energy-to-peak filtering for discrete-time nonlinear systems with measurement quantization

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ABSTRACT

This paper investigates the problem of robust energy-to-peak filtering for a class of discrete-time systems with norm-bounded uncertain parameters, measurement quantization and Lipschitz nonlinearity. Assume that the system measurement output is quantized by a static, memoryless and logarithmic quantizer before it being transmitted to the filter, while the quantization errors can be treated as sector-bound uncertainties. Attention is focused on the design of a robust energy-to-peak filter to mitigate quantization effects and ensure the filtering error system is asymptotically stable with a prescribed energy-to-peak noise attenuation level. Sufficient conditions for the existence of such a energy-to-peak filter are expressed in terms of linear matrix inequalities (LMIs). A numerical example is presented to demonstrate the effectiveness of the proposed design method.

1. Introduction

Recent years have witnessed a growing interest in investigating filtering problem of dynamic systems. This is mainly due to the fact that the state variables of systems are not always available [1,2]. It is well-known that the Kalman filtering approach is an important strategy for filtering problem of dynamic systems and has been applied in many practical applications [3]. However, Kalman filtering is based on two assumptions, i.e., the system model is assumed to be accurate and the priori information of the measurement noises is assumed to be precisely known, which cannot be always guaranteed in most of practical applications. Therefore, some new filtering approaches, such as H_∞ filtering [4,5], peak-to-peak filtering [6], and energy-to-peak filtering [7–21], are proposed to deal with model uncertainties as well as with unknown external noises.

Energy-to-peak filtering is an important filtering strategy in both filtering theory and practice, noticeable works include [7–21]. In [7], both reduced-order H_∞ and energy-to-peak filtering for linear systems have been concerned. Literature [8–12] focused on the problem of energy-to-peak filter design for polytopic uncertain linear systems. In [8], the energy-to-peak filtering problem for both continuous- and discrete-time systems with polytopic uncertainties have been considered. By using Projection lemma and parameter-dependent Lyapunov function approach, new conditions for energy-to-peak filters design of polytopic uncertain systems were proposed in [9]. Meng et al. [10] presented a novel parameter-dependent approach to deal with the energy-to-peak filtering of linear systems with polytopic uncertainties. Based on Fisher lemma, less conservative results on robust energy-to-peak filtering were obtained in [11]. Recently, by full using Fisher lemma and Projection lemma, further improved design conditions of robust full- and reduced-order energy-to-peak filters for polytopic uncertain discrete-time systems were given in [12]. In [13], the robust energy-to-peak filtering problem for uncertain stochastic time-delay systems has been addressed. The problem of robust energy-to-peak filtering for discrete-time switched polytopic linear systems with time-varying delay was investigated in [14]. Mahmoud and Xia [15] studied energy-to-peak filtering of uncertain discrete-time singular systems with time-varying delays. The energy-to-peak filtering problem for fuzzy stochastic systems with time-varying delays was considered in [16]. The study on energy-to-peak filter design for Markovian jump systems can be found in [17,18]. Energy-to-peak FIR filter design problem has been solved for linear systems in [19]. In [20], the problem of reduced-order energy-to-peak filtering for a class of nonlinear switched stochastic systems has been investigated. The problem of resilient energy-to-peak filtering for a class of uncertain continuous-time fuzzy systems was investigated in [21].

On the other hand, the study on networked control systems (NCSs) has attracted a growing interest in recent years. As pointed

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out in [22], the analysis and synthesis of networked control systems are quite different from the traditional control systems. Due to the limited communication capacity of the communication network, quantization [23–28], time-delays [29–32], and packet dropouts [30,33] are unavoidable in networked control systems. Among those mentioned phenomenons, quantization is an important issue in both networked control systems and digital signal processing systems. To mention a few, the problem of state estimation for linear discrete-time dynamic systems with quantized measurements has been addressed in [34], both infinite-level and finite-level quantizers were considered. In [35], the authors considered the problem of robust H_∞ filtering for a class continuous-time polytopic uncertain systems with limited communication constraints including data packet dropouts, signal transmission delay, and measurement quantization. The problem of H_∞ filtering for discrete-time T-S fuzzy systems with measurement quantization and packet dropouts has been studied in [36]. Han and Chang [37] investigated the problem of H_∞ filtering for discrete-time polytopic uncertain systems with measurement quantization. The variance-constrained state estimation problem has been considered for a class of networked multi-rate systems with network-induced probabilistic sensor failures and measurement quantization in [38]. In [39], the quantized H_∞ filtering problem for a class of discrete-time polytopic uncertain systems with packet dropout has been considered. To the authors' best knowledge, few attempts have been made on an robust energy-to-peak filter design for systems with measurement quantization, especially when the systems with Lipschitz nonlinearity, which motivates us for this study.

This paper focuses on the design of robust energy-to-peak filter for discrete-time systems subject to Lipschitz nonlinearity, norm-bounded uncertain parameters, and measurement quantization. The quantizer we consider in this paper is static, memoryless and logarithmic. Sufficient conditions for the existence of the robust energy-to-peak filter are obtained in terms of linear matrix inequalities (LMIs), which can not only mitigate the effects of quantization but also ensure the filtering error system is asymptotically stable with a prescribed energy-to-peak performance. Finally, we will illustrate the effectiveness of our main results by a numerical example.

Notations: The notations that are used throughout this paper are standard. $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ represent, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The notation $l_2[0,\infty)$ denotes the space of square-integrable vector functions over $[0,\infty)$. The notation $*$ is used to indicate the terms that can be induced by symmetry. Generally, for a square matrix A, A^T refers to its transpose and $He\{A\}$ denotes $(A+A^T)$.

2. Problem formulation

Consider the following uncertain nonlinear discrete-time systems:

$$\begin{aligned} x(k+1) &= (A+\Delta_A)x(k) + F\Gamma(k,x(k)) + Bw(k) \\ y(k) &= (C+\Delta_C)x(k) + Dw(k) \\ z(k) &= (L+\Delta_L)x(k) + Ew(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state variable; $y(k) \in \mathcal{R}^m$ is the measurement output; $z(k) \in \mathcal{R}^q$ is the signal to be estimated; $w(k) \in \mathcal{R}^v$ is the noise signal that is assumed to be the arbitrary signal in $l_2[0,\infty)$; $\Gamma(k,x(k)) \in \mathcal{R}^l$ denotes the nonlinear function of the system state $x(k)$; the matrices $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times v}$, $F \in \mathcal{R}^{n\times l}$, $C \in \mathcal{R}^{m\times n}$, $D \in \mathcal{R}^{m\times v}$, $L \in \mathcal{R}^{q\times n}$, $E \in \mathcal{R}^{q\times v}$ are known system matrices and the matrices $\Delta_A \in \mathcal{R}^{n\times n}$, $\Delta_C \in \mathcal{R}^{m\times n}$, and $\Delta_L \in \mathcal{R}^{q\times n}$ represent the time-varying norm-bounded parameter uncertainties that satisfy

$$\begin{bmatrix} \Delta_A \\ \Delta_C \\ \Delta_L \end{bmatrix} = \begin{bmatrix} X_A \\ X_C \\ X_L \end{bmatrix} \Delta_x(k) Y \tag{2}$$

where X_A, X_C, X_L, and Y are known constant matrices of appropriate dimensions, and $\Delta_x(k)$ is unknown matrix satisfying

$$\Delta_x^T(k)\Delta_x(k) \le I \quad \forall\ k. \tag{3}$$

The parameter uncertainties Δ_A, Δ_C, and Δ_L are said to be admissible if (2) and (3) hold.

Assumption 1. We shall assume that system (1) is locally Lipschitz with respect to the state $x(k)$ in a region $\mathcal{D}$ containing the origin, i.e.

$$\begin{aligned} &\Gamma(0,x(0)) = 0, \\ &\|\Gamma(k,x_1(k)) - \Gamma(k,x_2(k)))\| \\ &\quad \le \|\mathcal{G}(x_1(k) - x_2(k))\|, \forall\ x_1(k), x_2(k) \in \mathcal{D}, \end{aligned} \tag{4}$$

where $\mathcal{G}$ denotes the Lipschitz real matrix of $\Gamma(k, x(k))$ with appropriate dimensions.

For nonlinear system (1), we consider a filter described by the following form

$$\begin{aligned} x_f(k+1) &= A_f x_f(k) + B_f f(y(k)) \\ z_f(k) &= C_f x_f(k) + D_f f(y(k)) \end{aligned} \tag{5}$$

where $x_f(k) \in \mathcal{R}^n$ is the state of the filter and $z_f(k) \in \mathcal{R}^q$ is the output of the filter; $f(y(k)) \in \mathcal{R}^m$ is the quantized measurement; the matrices $A_f \in \mathcal{R}^{n\times n}$, $B_f \in \mathcal{R}^{n\times m}$, $C_f \in \mathcal{R}^{q\times n}$, $D_f \in \mathcal{R}^{q\times m}$ are the filter gain matrices to be designed.

The quantizer we consider is logarithmic static and time-invariant quantizer given by Fu and Xie [24]. The set of quantized levels is described by $U^{(j)} = \{\pm u_i^j, u_i^j = (\rho^j)^i u_0^j, i = \pm 1, \pm 2, \cdots\} \cup \{\pm u_0^j\} \cup \{0\}, u_0^j > 0,\ 0 < \rho_j < 1$ where the parameter ρ_j denotes the quantization density and the quantizer $f_j(\cdot)$ is defined as follow:

$$f_j(y) = \begin{cases} v_i^{(j)} & 0 \le \dfrac{v_i^{(j)}}{(1+\delta_j)} < y \le \dfrac{v_i^{(j)}}{(1-\delta_j)} \\ 0 & y = 0 \\ -f_j(-y) & y < 0 \end{cases} \tag{6}$$

$$\delta_j = \frac{1-\rho_j}{1+\rho_j} \tag{7}$$

Then, by using the sector bound method described in [24], we can get that for any $y(k)$,

$$f(y(k)) - y(k) = \Delta(k)y(k) \tag{8}$$

where $\Delta(k) = diag\{\Delta_1(k), \Delta_2(k), \ldots, \Delta_m(k)\},\ |\Delta(k)| \le \delta$.

Combining (1) and (4)–(7), defining $\xi(k) = [x^T(k),\ x_f^T(k)]^T$ and $e(k) = z(k) - z_f(k)$ the filtering error system can be given as follow:

$$\begin{aligned} \xi(k+1) &= \tilde{A}\xi(k) + \tilde{F}\Gamma(k,\xi(k)) + \tilde{B}w(k) \\ e(k) &= \tilde{C}\xi(k) + \tilde{D}w(k) \end{aligned} \tag{9}$$

where $\Gamma(k,\xi(k)) = [\begin{smallmatrix}\Gamma(k,x(k)) \\ \Gamma(k,x_f(k))\end{smallmatrix}]$ and

$$\tilde{A} = \begin{bmatrix} A+\Delta_A & 0 \\ B_f(I+\Delta(k))(C+\Delta_C) & A_f \end{bmatrix},$$

$$\tilde{B} = \begin{bmatrix} B \\ B_f(I+\Delta(k))D \end{bmatrix},\ \tilde{F} = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} L+\Delta_L - D_f(I+\Delta(k))(C+\Delta_C) & -C_f \end{bmatrix},$$

$$\tilde{D} = E - D_f(I+\Delta(k))D.$$