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Sparse analysis model based multiplicative noise removal with enhanced regularization☆

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The multiplicative noise removal problem for a corrupted image has recently been considered under the framework of regularization based approaches, where the regularizations are typically defined on sparse dictionaries and/or total variation (TV). This framework was demonstrated to be effective. However, the sparse regularizers used so far are based overwhelmingly on the synthesis model, and the TV based regularizer may induce the stair-casing effect in the reconstructed image. In this paper, we propose a new method using a sparse analysis model. Our formulation contains a data fidelity term derived from the distribution of the noise and two regularizers. One regularizer employs a learned analysis dictionary, and the other regularizer is an enhanced TV by introducing a parameter to control the smoothness constraint defined on pixel-wise differences. To address the resulting optimization problem, we adapt the alternating direction method of multipliers (ADMM) framework, and present a new method where a relaxation technique is developed to update the variables flexibly with either image patches or the whole image, as required by the learned dictionary and the enhanced TV regularizers, respectively. Experimental results demonstrate the improved performance of the proposed method as compared with several recent baseline methods, especially for relatively high noise levels.

1. Introduction

Multiplicative noise, also known as speckle noise, is often observed in synthetic aperture radar (SAR) and sonar (SAS) images, due to the effect of interference introduced in their acquisition processes [1]. Compared to additive Gaussian noise often assumed in traditional image denoising, removing speckle noise is deemed to be more difficult for two reasons. Firstly, the noise is multiplied with (rather than added to) the original image, which usually degrades the images more severely as compared with additive noise [2]. Secondly, the study of the statistical properties of speckle noise indicates that Gamma and Rayleigh distributions are more suitable for modelling such noise [1–4] instead of the widely used Gaussian distribution in conventional image denoising, and thus the data fidelity term derived from the noise model is not quadratic, raising difficulties for optimization.

Mathematically, the observed image $\mathbf{w} \in \mathbb{R}^N$ (reshaped from a $\sqrt{N} \times \sqrt{N}$ image) contaminated by the speckle noise $\mathbf{u} \in \mathbb{R}^N$, can be represented as [4,5]

$$\mathbf{w} = \mathbf{g} \circ \mathbf{u}, \tag{1}$$

where $\mathbf{g} \in \mathbb{R}^N$ denotes the image to be restored. The symbol $\circ$ denotes the Hadamard product (i.e. entry-wise product) of two matrices/vectors. The aim of despeckling is to estimate $\mathbf{g}$ from the observed image $\mathbf{w}$. In this paper, we focus on Gamma distributed multiplicative noise, such that the elements of $\mathbf{u}$ are assumed to be independent and identically distributed (i.i.d.) with probability density function given by [2,4,5]

$$f_u(u) = \frac{L^L}{\Gamma(L)} u^{L-1} e^{-Lu}, \tag{2}$$

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where L is a positive integer defining the noise level and $\Gamma(\cdot)$ is the classical Gamma function given by $\Gamma(L) = (L-1)!$. A smaller L indicates stronger noise.

1.1. Related work

Classical methods for removing multiplicative noise are spatial filtering [6–8] and wavelet domain filtering [9,10]. More recently, regularization based approaches to denoising, where the image reconstruction task is formulated as an optimization problem with regularizers, have attracted much attention [4,5,11–13]. A popular regularizer employed in these approaches is total variation (TV) which was proposed originally for reducing additive Gaussian noise [14]. The TV-based methods were then used for multiplicative noise in the original image domain as in Eq. (1) or in the log-domain by applying a logarithmic transform. Typical examples performed in the original domain are the first TV-based multiplicative noise removal method proposed in [15] and the method of Aubert–Aujol (AA) [11]. The method in [15] minimizes the TV of the image to be recovered with the constraints exploiting the mean and variation of the noise, but this method is not effective for removing Gamma distributed noise as the noise considered in its restoration model is assumed to follow a Gaussian distribution. The AA method [11] exploits a Bayesian *maximum a posteriori* (MAP) estimate, yielding an image restoration model consisting of a data fidelity term based on the prior distribution of the multiplicative noise and a TV regularization term. However, the quality of the image restored by the AA method may be limited by the local solutions obtained from the optimization of a non-convex model. Another class of denoising methods based on the TV regularizer considers the image restoration in the log-domain [4,5,12,13], aiming to simplify the multiplicative noise model as an additive model which is easier to deal with than the original model. In general, the reconstruction models employed in these methods commonly consist of a data fidelity term and regularization terms reflecting prior information related to the image. However, the formulations of these terms and optimization approaches may differ substantially. In [12], Shi and Osher (SO) considered both the data fidelity and TV terms of the AA method [11] in the log-domain to overcome the non-convex optimization issue. Multiplicative Image Denoising with the Augmented Lagrangian (MIDAL) algorithm [4] uses the same model as used by SO but applies a different optimization framework based on variable splitting and augmented Lagrangian for better numerical efficiency. Apart from the data fidelity term and the TV regularization as in the reconstruction model used by SO [12] and the MIDAL algorithm [4], the method presented in [13] also incorporates a quadratic data fitting term to apply the TV term in a more efficient manner, but it tends to be outperformed by the MIDAL algorithm [4].

Although the TV regularization proves to be effective for reducing multiplicative noise, the smoothly varying regions in the original image are usually recovered as piecewise constant areas, which is also well known as the stair-casing effect [2]. An approach to avoid this issue is to introduce priors on the image to be recovered. Recently, the sparsity prior was shown to be helpful for the reconstruction of images with multiplicative Gamma noise [2,5,16]. Duran, Fadili and Nikolova (DFN) [2] adopted the sparsity prior by considering the sparsity of the image in the curvelet transformed domain and restoring the frame coefficients via a TV regularized formulation in the log-domain. As dictionaries learned from the related data have the potential to fit the data better than pre-defined dictionaries, dictionary learning techniques in sparse representation have also been utilized to model the sparsity prior [5,16]. The methods proposed in [16] and [5] both introduce dictionary learning to the TV regularized model [4,12], but with different frameworks. These two methods are referred to as MNR-DL-TV-1 (Multiplicative Noise Removal via Dictionary Learning and Total Variation) [16] and MNR-DL-TV-2 [5] respectively. In these two methods, the dictionary is learned by the K-SVD algorithm [17] which is a well-known dictionary learning method based on the sparse synthesis model. The MNR-DL-TV-1 method performs noise reduction in two stages: the image is first denoised using the learned dictionary; and then a model based on an ℓ_2 data fidelity term and TV regularization is applied to further improve the denoising result. In contrast, the MNR-DL-TV-2 method formulates the image reconstruction task as an optimization problem containing two regularizers: a learned dictionary based term and a TV term. However, we have found that the performance of MNR-DL-TV-2 is limited for relatively high noise-levels, as shown in our simulations (see Section 5.1 later).

It should be noted that the learned dictionaries employed in the MNR-DL-TV-1 [16] and MNR-DL-TV-2 [5] methods are both based on the sparse synthesis model [17]. In recent years, the sparse analysis model, as a counterpart of the synthesis model, has attracted much attention [18,19]. Dictionary learning based on the sparse analysis model was also shown to be effective in the reduction of additive Gaussian noise [20], [21], however, few researchers have studied its potential for removing multiplicative noise. We have proposed a speckle noise removal method in [22] which applies the dictionary learned based on the analysis model to the regularizer of the restoration formulation. This approach, referred to as Removing Speckle Noise via Analysis Dictionary Learning (RSN-ADL), has the ability to preserve details while reducing multiplicative noise, however the smooth regions are not well-recovered, as will be illustrated in Section 5.

1.2. Contributions

In this paper, we propose a new model for reconstructing the image from a multiplicative noise corrupted image and develop a novel method for optimizing this model. The proposed method applies a sparse analysis model based regularizer and a smoothness regularizer. The joint employment of these two regularizers, which is different from the existing methods, aims to exploit the benefits of both priors and partly addresses the limitations of the existing methods mentioned above. Specifically, the sparse analysis model based regularizer is constructed with an analysis dictionary learned from image patches via the Analysis SimCO algorithm [21,23], and the smoothness regularizer is formed based on the pixel-wise differences in the horizontal and vertical directions. This reconstruction model extends our previous work [22] by introducing the smoothness regularization term. Since the dictionaries used in the regularizer of [22] are usually well adapted to textures but not for smooth areas [5], the introduction of the smoothness regularizer in the proposed model has the potential to overcome this issue. Compared with the methods based on TV regularization, for example the MIDAL algorithm [4], the proposed model can mitigate the stair-casing effect appearing in the recovered images due to the application of the analysis model based regularization, as will be demonstrated in Section 5. The proposed model also shows advantages for a relatively high level of noise, compared with the DFN [2] and MNR-DL-TV-2 [5] algorithms.

The introduction of the two regularizers in our restoration formulation, however, renders the optimization task non-trivial, especially since the two regularizers are defined from different representations of the image. In particular, the dictionary is learned with image patches instead of the whole image in order to reduce the computational complexity. As a result, the sparse analysis model based regularizer is represented with image patches. The smoothness regularizer, on the other hand, is defined with pixel-wise differences calculated across the whole image. In order to address the optimization of the presented model, we propose a