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Entropy-based bilateral filtering with a new range kernel

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ABSTRACT

Bilateral filter (BF) is a well-known edge-preserving image smoothing technique, which has been widely used in image denoising. The major drawback of BF is that its range kernel is sensitive to noise. To address this issue, we propose an entropy-based BF (EBF) with a new range kernel which contains a new range distance. The new range distance is robust to noise by exploiting the information from the denoised estimate and the corresponding method noise, i.e., the difference between the noisy image and its denoised estimate. Moreover, in order to consider the local statistics of images, local entropy is applied to adaptively guide the range parameter selections. This allows our method to adapt to the images with different characteristics. Experimental results demonstrate that the proposed EBF significantly outperforms the standard BF in terms of both quantitative metrics and subjective visual quality.

1. Introduction

Bilateral filter (BF) [1] is a well-known edge-preserving tool, which has been widely used in image denoising. To remove noise while preserving edges, BF uses the weighted average of nearby pixels in a local neighborhood, where weights rely on the spatial and intensity distance. The output of BF centered at **q** can be expressed as

$$\widehat{y}(\mathbf{q}) = \frac{\sum_{\mathbf{p}\in\mathcal{S}} w_{\sigma_s}\cdot w_{\sigma_r}\cdot y(\mathbf{p})}{\sum_{\mathbf{p}\in\mathcal{S}} w_{\sigma_s}\cdot w_{\sigma_r}}, \tag{1}$$

where $y(\mathbf{p})$ is the noisy pixel, and $\mathcal{S}$ is the neighborhood of size $(2r+1)\times(2r+1)$ centered at $\mathbf{q}$; w_{σ_s} and w_{σ_r} are the spatial kernel and range kernel, both of which determine the practical performance of BF. More precisely,

$$w_{\sigma_s} = \exp\left(-\frac{||\mathbf{p}-\mathbf{q}||_2^2}{2\sigma_s^2}\right), \tag{2}$$

where the spatial distance $||\mathbf{p}-\mathbf{q}||_2^2$ measures the spatial correlations and the spatial parameter σ_s controls the size of the spatial neighborhood, and

$$w_{\sigma_r} = \exp\left(-\frac{|y(\mathbf{p})-y(\mathbf{q})|^2}{2\sigma_r^2}\right), \tag{3}$$

where the range distance $|y(\mathbf{p})-y(\mathbf{q})|^2$ measures the intensity correlations and the range parameter σ_r controls how much a nearby pixel is weighted due to the pixel intensity.

The denoising performance of BF is mainly determined by the range kernel rather than the spatial kernel, which was demonstrated in [2]; hence we focus hereafter on the improvement of the range kernel. As stated above, the range kernel contains two crucial factors, i.e., the range distance and range parameter. However, both of these factors are sensitive to noise. Thus many research efforts have concentrated on how to obtain a good estimation of these two factors under various noise levels.

The conventional range distance is computed directly from noisy images. This, however, leads to large estimation bias due to the seriously corrupted correlations of pixels under strong noise. Some invariants [3–6] of BF attempted to alleviate the estimation bias by calculating the range distance from denoised images. However, these methods still cannot achieve satisfying results under strong noise, since the denoised images are usually far away from the original ones. Moreover, from the analysis of method noise[1] [7], the denoised image does not contain the complete details. In other words, there still exist the *residual image structures* (the original image structures) in method noise. As a result, the estimation accuracy of the range distance can be further improved, if we can exploit the local similarities of the residual image structures.

Besides, many research works focus on tuning the range parameter. Some recently developed adaptive bilateral filters (ABFs) [2,8,9] have adapted the range parameter to the global [2,8] or local structures of the images [9]. Among them, Zhang et al. [2] demonstrated that the range parameter has more impact on the denoising performace than the spatial parameter, and showed that

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[1] Method noise is often defined as the difference between the noisy image and its corresponding denoised image.

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the optimal range parameter is linearly proportional to the standard deviation of the noise, i.e., $\sigma_r = k \cdot \sigma$, where k is a fixed value chosen empirically. Using such globally fixed range parameter may lead to unsatisfying results for the images with various structures, such as *Barbara*, since the global range parameter cannot consider the local structures. More recently, a new ABF [9] with spatially adaptive parameter selections has been proposed, which, however, requires high computational complexity. Therefore, it is still demanding to propose a novel BF with spatially adaptive parameter selections and low complexity.

Motivated by the above observations, we propose an adaptive entropy-based BF (EBF) with a new range kernel which includes a new range distance. To be specific, the new range distance is estimated from a "clean" image, which is derived by exploiting the information both from the denoised estimate and the residual image in method noise. Compared with the range distance estimated from the noisy or denoised image, ours is more robust to various noise levels. Furthermore, in order to consider the structural characteristics of the images, local entropy serves as a guide for adaptive range parameter selections. In information theory, local entropy represents the variance of local regions and catches the natural properties of transition regions of edges. Based on this fact, our method builds a set of entropy-based local image descriptors, extracted from the noisy image and used to modulate the range parameter across the image. Unlike the above-mentioned methods [2,8,9] which learn the optimal filter parameters with high complexity, our method obtains adaptive range parameters at a local scale with a relatively low complexity. To apply the proposed EBF for image denoising, a two-stage EBF based framework is presented, which is detailed in Fig. 4. In summary, the main contributions of the paper are as follows:

1. A new range distance is estimated from a "clean" image, which exploits the information from the denoised image and the residual image in method noise.
2. A simple but effective approach is proposed to adaptively tune the range parameter, which applies local entropy to characterize the local structures of images.

The rest of this paper is organized as follows. Section 2 briefly reviews the related works, including the major image denoising methods and the existing progress of bilateral filter. In Section 3, we introduce the basic concepts of method noise and local entropy. In Section 4, we propose an EBF-based denoising framework. Section 5 shows the experimental results. Finally, we draw the conclusions.

2. Related works

In general, image denoising methods can be divided into three categories: spatial domain, transform domain and learning-based denoising methods [10], where BF belongs to spatial domain methods. In this section, we briefly review the major methods for image denoising and the main previous works related to BF.

Spatial domain methods attempt to utilize the correlations of natural images [11]. According to the selection of pixels (patches), spatial filters can be categorized as local and nonlocal filters. Local filters are restricted in a local spatial distance, such as Gaussian filtering, anisotropic filtering [12], total variation minimization (TV) [13,14] and joint filtering [15]. However, these methods cannot perform well at high noise levels because the correlations between neighboring pixels are corrupted by the severe noise. To overcome this issue, the nonlocal filters utilize the self-similarity of natural images in a nonlocal manner. Nonlocal means (NLM) filter [7], achieves a denoised pixel by weighted averaging all other pixels in the noisy image, whose pixel similarity depends on the patch. The main drawback of NLM filters is that these patch-based methods are computational-intensive and often tend to over-smooth image details. More recently, the idea of nonlocal similarity has been extended to transform domain [16–18] and learning-based methods [19–21] in order to further improve the denoising performance. Among them, learning-based method proposed by Elad et al. [19] obtained good results based on sparse and redundant representations over learned dictionaries. Besides, the so-called BM3D [16] achieved remarkable results by combining the patch-based techniques like NLM with transform-based filtering. Beyond utilizing the nonlocal prior, some important works can also obtain remarkable results by utilizing low-rank prior of images, such as WNNM [22]. In a different direction, it was observed in [23,24] that neural networks can be successfully applied to image denoising.

Besides the above patch-based methods, BF has received much attention due to its simplicity and efficiency. Most BF-based methods can be roughly divided into two lines of work, i.e. theoretical analysis and performance improvement.

Some theoretical works of BF deserve mentioning. In [25], it was demonstrated that BF emerges from Bayesian approach and is identical to the first iteration of Jacobi algorithm. Barash et al. [26] related BF with anisotropic diffusion (AD) [12]. Besides, the relationship between BF and TV regularization was developed in [27], which was further generalized by casting BF, median filters, mode filtering, nonlinear diffusion filtering, and regularization techniques in a single unified framework of discrete regularization theory in [28]. In a different direction, Takeda et al. [29] observed that BF is a simple example of kernel regression. Recently, Caraffa et al. [30] proposed an iterated version of BF that is robust to outliers and demonstrated how it can be used to remove non-Gaussian noise.

More works focus on the performance improvement of BF, including parameter selection and acceleration. For parameter selection, Zhang and Gunturk [2] demonstrated that the optimal σ_s is relatively insensitive to the noise standard deviation σ and it is generally in the range [1.5, 2.1], while the range parameter σ_r has more impact on the denoising performance. Based on the experimental results obtained on a large set of natural images, Zhang and Gunturk suggested that the optimal σ_r should be approximately linearly related to σ. Another ABF [8] for sharpness enhancement and noise removal used a complex training procedure to optimize the filter parameters. In addition, many works have been done to accelerate BF. A direct computation of BF requires $O(r^2)$ operations per pixel. To speed up BF, researchers have come up with several fast algorithms [31–37]. Durand et al. [31] sped up BF based on a piecewise-linear approximation in the intensity domain and subsampling in the spatial domain. In addition, it was observed in [32] that BF can be considered as a linear filter acting in three-dimensions, where the three-dimensions are obtained by augmenting the image intensity to the spatial dimensions. The algorithm in [33] enabled bilateral filtering in constant time $O(1)$ without sampling, which was further improved by using trigonometric range kernels in [34,35]. More recent works like [36,37] further accelerated BF by approximating the range kernel using polynomial and trigonometric functions.

3. Fundamentals

3.1. Method noise

Given a gray-level image $\mathbf{y}$ contaminated with additive white Gaussian noise (AWGN), i.e.,

$$\mathbf{y} = \mathbf{x} + \mathbf{n}, \tag{4}$$

where $\mathbf{x}$ is the clean image, and $\mathbf{n}$ is AWGN with zero mean and standard deviation σ.