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Adaptive detection using both the test and training data for disturbance correlation estimation

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ABSTRACT

This paper examines a target detection problem in colored Gaussian disturbance with an unknown covariance matrix. In many classic adaptive detectors, the covariance estimator is formed by using only the training data. This necessitates calculating a new covariance estimator for each cell under test (CUT) during the cell-by-cell target search process. We consider herein an alternative approach that forms the covariance matrix estimate by using both test and training data for detection in homogeneous environments. This approach is computationally much more efficient since the covariance matrix estimator is computed only once and can be applied for target detection at each CUT. Using this estimator, we propose a new detector with two tunable parameters, which includes several existing detectors as special cases. Closed-form expressions for the probabilities of false alarm and detection are derived in the matched and mismatched cases for both non-fluctuating and fluctuating target models. Simulation results reveal that the rejection capability of mismatched signals of the proposed detector can be flexibly controlled by adjusting its tunable parameters. In particular, the proposed detector can achieve the same detection performance as the generalized likelihood ratio test (GLRT) detector derived by Kelly, but has a much lower computational burden.

1. Introduction

Target detection in Gaussian disturbance with unknown covariance matrix has been a topic of long-standing interest in radar/sonar signal processing [1–19]. Typically, the presence of target is sought in a (range) cell under test (CUT). The data collected from the CUT is referred to as the test (primary) data. A set of independent and identically distributed training (secondary) data samples, which contain disturbance only, are employed to estimate the unknown disturbance covariance matrix. In radar practice, these training data samples are usually collected from range cells adjacent to the CUT.

Several classic detection algorithms have been proposed in the past. Specifically, Kelly proposed a generalized likelihood ratio test (GLRT) detector through replacing all unknown parameters with their maximum likelihood (ML) estimates under each hypothesis in one step [1]. Meanwhile, an adaptive matched filter (AMF) detector was derived with an *ad hoc* two-step procedure [3]. In particular, it first assumes the disturbance covariance matrix is known and obtains a GLRT by maximizing over other unknown parameters; then a test statistic is obtained by substituting the disturbance covariance matrix with its ML estimate based on the training data alone. In [4], an adaptive coherence estimator (ACE) was proposed to handle a non-homogeneity between the test and training data. A prominent feature of the above three detectors is that they all achieve constant false alarm rate (CFAR) with respect to the unknown disturbance covariance matrix. Note that the performance of the GLRT, AMF and ACE cannot be flexibly adjusted. In the last two decades, researchers have proposed many tunable detectors including parametric [10] and two-stage receivers [20–23].

Since the location of the target to be detected is generally unknown in practice, a grid search is often resorted to, which divides the desired radar surveillance area into many (range) cells or bins. We need to test each cell one by one to decide whether the interested target is present or not. For target detection in a specific cell, a standard approach is to employ the data collected from cells adjacent to the CUT as training data, and then use these training

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data to estimate the disturbance covariance matrix. Obviously, the training data are different for different CUTs, which implies that a new estimate of the disturbance covariance matrix has to be calculated for a distinct CUT in the detectors mentioned above. This operation entails heavy computational complexity, particularly in space-time adaptive processing [24,25], where the data dimension is the product of the number of array elements and the number of taps of the Doppler filters which can be quite high even with a moderate number of array antennas and filter taps.

For target detection in homogeneous environments, Gerlach in [26] introduced a new covariance matrix estimator to avoid calculating a large number of covariance matrices and their inverses by employing both the test and training data. Note that the idea of using the whole data block for estimation is similar to the mean level adaptive detector (MLAD) with scalar data [27,28]. Once the disturbance covariance matrix estimate is calculated with the whole data block, it can be applied for detection in each cell. Apparently, this approach is computationally much more efficient. It should be pointed out that the whole data might contain a target signal, which can lead to some performance loss, but at the benefit of significantly reducing the computational complexity. The performance loss is considered negligible when the training size is sufficiently large and targets are rare.

Based on the covariance estimator using both the test and training data, we propose in this paper a new detector with two tunable parameters a and b, which includes the detector in [26] as a special case. In particular, the proposed detector with $a = 1$ and $b = -1$ provides the same detection performance as Kelly's GLRT detector. It should be emphasized that the proposed detector has a much lower computational burden than the conventional detectors (i.e., Kelly's GLRT, the AMF, and the ACE). The statistical properties of the proposed detector are investigated for both the matched and mismatched cases depending on whether the actual steering vector is aligned with the nominal one. It should be pointed out that the mismatched case is not studied in [26]. Closed-form expressions for the probabilities of false alarm and detection of the proposed detector are derived for both non-fluctuating and fluctuating target models. In the non-fluctuating model, the target amplitude is considered to be deterministic, while in the fluctuating model, the target amplitude is assumed to have a generalized Chi distribution which includes the Rayleigh distribution as a special case. These theoretical results are confirmed by using Monte Carlo (MC) simulations. Numerical results demonstrate that the selective capability of the proposed detector can be flexibly adjusted by changing the tunable parameters.

The remainder of this paper is organized as follows. Section 2 formulates the problem to be studied. In Section 3, a detector with tunable parameters is proposed, and performance analysis is provided in detail. Simulation results are illustrated in Section 4 and finally the paper is summarized in Section 5.

Notation. Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^\dagger$ denote transpose, complex conjugate and complex conjugate transpose, respectively. The notation $\sim$ means "is distributed as," and $\mathcal{CN}$ denotes a circularly symmetric, complex Gaussian distribution. $\overset{\mathrm{d}}{=}$ means the former and latter random quantities have the same distribution. χ_n^2 denotes a real central Chi-squared distribution with n degrees of freedom, while $\chi_n'^2(\zeta)$ denotes a real non-central Chi-squared distribution with n degrees of freedom and a non-centrality parameter ζ. $|\cdot|$ represents the modulus of a complex number and $\jmath = \sqrt{-1}$. $\mathrm{C}_n^m = \frac{n!}{m!(n-m)!}$ and $\Gamma(\cdot)$ are the binomial coefficient and the Gamma function, respectively.

2. Data model

Consider the following model of the received data in a CUT:

$$\mathbf{x} = \alpha\,\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{s}$ is a known steering vector of dimension $N \times 1$; α is a deterministic but unknown complex scalar accounting for the target reflectivity and the channel propagation effects; the disturbance $\mathbf{n}$ is assumed to have a circularly symmetric, complex Gaussian distribution, i.e., $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, where $\mathbf{R}$ is a positive definite covariance matrix of dimension $N \times N$. These data may be temporal samples, spatial samples (obtained with an array), or any mix of the above.

In practice, the disturbance covariance matrix $\mathbf{R}$ is usually unknown. To estimate it, we impose a standard assumption that there exists a set of homogeneous secondary data free of target signal components, i.e., $\{\mathbf{y}_k | \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}), k = 1, 2, \ldots, K \text{ and } K \geq N\}$. In array signal processing, this set of secondary data are usually collected from the range cells adjacent to the CUT. Let the null hypothesis (H_0) be that the target signal is free in the test data and the alternative hypothesis (H_1) be that the test data contain the target signal. Hence, the detection problem is to decide between the null hypothesis

$$H_0 : \begin{cases} \mathbf{x} \sim \mathcal{CN}(\mathbf{0},\ \mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0},\ \mathbf{R}),\ \ k = 1, \ldots, K, \end{cases} \tag{2a}$$

and the alternative one

$$H_1 : \begin{cases} \mathbf{x} \sim \mathcal{CN}(\alpha\mathbf{s},\ \mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0},\ \mathbf{R}),\ \ k = 1, \ldots, K. \end{cases} \tag{2b}$$

It is easy to show that based on these secondary data, the ML estimate of the disturbance covariance matrix (up to a scaling factor) is

$$\hat{\mathbf{R}} = \sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^\dagger. \tag{3}$$

Using this disturbance covariance matrix estimate, several classic adaptive detectors were proposed, including, e.g., the GLRT [1], AMF [3], and ACE [4]:

$$T_{\mathrm{GLRT}} = \frac{|\mathbf{s}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{x}|^2}{(\mathbf{s}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{s})(1 + \mathbf{x}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} t_{\mathrm{GLRT}}, \tag{4}$$

$$T_{\mathrm{AMF}} = \frac{|\mathbf{s}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{x}|^2}{\mathbf{s}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{s}} \underset{H_0}{\overset{H_1}{\gtrless}} t_{\mathrm{AMF}}, \tag{5}$$

$$T_{\mathrm{ACE}} = \frac{|\mathbf{s}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{x}|^2}{(\mathbf{s}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{s})(\mathbf{x}^\dagger \hat{\mathbf{R}}^{-1}\mathbf{x})} \underset{H_0}{\overset{H_1}{\gtrless}} t_{\mathrm{ACE}}. \tag{6}$$

In the applications of the above three detectors, we need to calculate a new covariance estimator for a different CUT in the grid search stage, which incurs heavy computational burdens, especially when the data dimension is high (e.g., in space-time adaptive processing [24,25]), and/or the number of cells to be tested is large (e.g., in high-resolution radar).

3. Detector with tunable parameters

To alleviate the computational burden stated above, we estimate the disturbance covariance matrix by using both the test and training data, i.e.,

$$\tilde{\mathbf{R}} = \hat{\mathbf{R}} + \mathbf{x}\mathbf{x}^\dagger. \tag{7}$$