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A note on sensor arrangement for long-distance target localization

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ABSTRACT

We considered a sensor formation problem for target localization based on the time difference of arrival (TDOA) data. It is assumed that the target is located far off from the sensors while the sensors are relatively close to each other. The estimators of the target location expressed in terms of the spherical coordinates are found to be uncorrelated when the sensors are arranged in a concentric ring formation. The proposed optimal formation of sensors is in the concentric ring formation and is shown to change in accordance with sensors' angular positions with respect to the line-of-sight vector from the reference sensor to the target. The proposed optimal sensor formations are compared in the context of estimation accuracy both in theory and by numerical experiments. They are shown optimal in general through refined numerical experiments.

1. Introduction

The target localization problem is found in a wide range of situations in various scales. A few examples include the global positioning system (GPS) using satellite communication, the passive tracking of a signal-emitting military target, and locating technologies of emergency callers. Various localization methods of different physical natures have been developed and studied. The time difference of arrival (TDOA) method uses the measured time differences of the signals received by a set of sensors, and the frequency difference of arrival (FDOA) method utilizes the frequency Doppler shift phenomenon which arises when there exists a relative movement between the source and the receiver. The angle of arrival (AOA) method makes use of the difference in the direction to the target from the sensors at different locations.

Analysis of the localization problem based on different types of measurement is abound in the literature [4,8,6,1,14,17,20,19,22], and algebraic equations characterizing the problem have been derived [13,18,12,5]. Several methods for finding their solutions are suggested and their stochastic properties are also discussed. The total least square estimation [21] and the Taylor series expansion [7] are among the most commonly used techniques for TDOA localization.

The studies on the localization problem have often focused on formulating estimating equations, which are functions of the target location and the measurement vectors, and on suggesting ways of finding their algebraic solutions. Difficulty in localization is due to the nonlinearity of the estimating equations. Cheung et al. [4], among others, formulated the estimation problem as a least squares problem with constraints by transforming the quadratic distance relations into linear matrix identities with redundant parameters. The performance of this method was discussed by analyzing the approximate variances of the estimates obtained by applying the method used for unconstrained least squares estimators.

Optimal sensor arrangement is crucial for performance improvement in the localization problem. In addressing this optimality issue, the geometric nature of the localization problem and dependence of tracking performance on sensor formation are incorporated into a formula which can be interpreted in terms of information criterion. Optimal sensor arrangements are found among others in [2,9,11,15,22,23] based on several types of measurements such as AOA, time-of-arrival, TDOA, received-signal-strength measurements. One of the results common among these works is that the optimal arrangement is in the form of a regular polygon with the target at its center. We can see, in these works, variations in the optimal sensor arrangement in accordance with some constraints on the measurement method or the purpose of the problem.

In this paper, we will consider a TDOA based localization problem where the sensors are not to be disclosed to the target and the target is far away from the sensor network while the sensors are arranged within a limited region at the other distant end of the target. The sensors consist of one main sensor and several auxiliary ones. All the sensors receive signal from the target in a passive mode and the main sensor

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collects data from the other sensors. The main sensor then computes the sensor positions and the range differences between the main and the other sensors. In our target localization problem, the uniform angular array (UAA) surrounding the target as in [15,22,23] is not considerable and the solutions of the estimating equations tend to be increasingly sensitive to the sensor formation. We will address this problem from the perspectives of information optimization and sensor-target geometry. This work is disjunctive from preceding works in the sense that we provide a more general mathematical framework to deal with the optimization problem of multi-sensor formation for 3D target localization when the target is at a good distance from the sensors. The optimal formations are compared using the mean squared error (MSE). It is also shown that the optimal sensor arrangements change in accordance with sensors' angular positions with respect to the line-of-sight vector from the main sensor to the target. In this sense, we regard the main sensor as a reference for position coordinates.

This paper is organized in eight sections. In Section 2, we describe briefly a target-sensor geometry on a plane and it is extended to a 3D problem in Section 3, where we introduce a probability model of measurements of range differences which are made by the TDOA method. We then derive the formula of the Fisher information of the target location under the assumption that the distance between target and sensor is much larger than that between sensors. We found in Section 5 that the estimators of the target distance (d_t) and the two types of the target angles, inclination (ϕ_t) and azimuth (θ_t), are uncorrelated when the sensors are arranged in a UAA on a ring. We also found the sensor formations which minimize the MSE for the three parameters, d_t, ϕ_t, and θ_t. This result is applied, in Section 6, in search of optimal sensor formations based on the MSE of the estimators. We ran numerical experiments in Section 7 to show a detailed comparison among sensor formations and that the proposed sensor formation is optimal in general. Section 8 concludes the paper with some concluding remarks.

2. Geometric preliminaries on a plane

Suppose there are two sensors 0 and i with sensor 0 at the origin and denote the distance between the two sensors by d_i and that between sensor i and target t by d_{it}. We define

$$r_i = d_{0t} - d_{it}$$

and call the r_i the range difference between the two sensors, sensors 0 and i. As for the angle, we denote by θ_t the angle at the origin measured counter-clockwise between the reference line (i.e., the horizontal line) and the target line which is the line connecting the origin (sensor 0) and the target and call it target angle; by θ_i the angle between the reference line and sensor i. For the sake of convenience, we will write d_t instead of d_{0t}. The geometry of the three objects is given in Fig. 1.

r_i can be expressed in terms of d_t, θ_t, d_i, and θ_i as

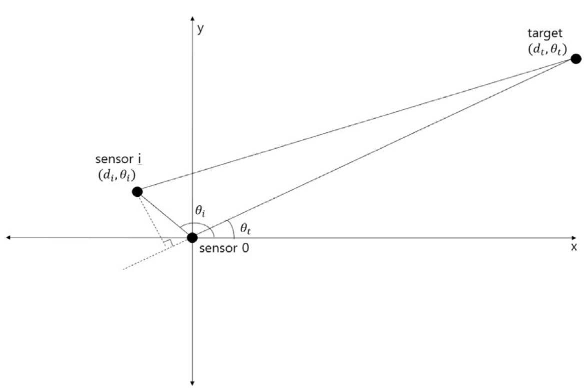

Fig. 1. Sensor-target geometry on a plane.

$$r_i = d_t\left(1 - \sqrt{1 + \frac{d_i^2}{d_t^2} - 2\frac{d_i}{d_t}\cos(\theta_i - \theta_t)}\right)$$

which is immediate from the law of cosines. We can see in this equation that r_i is a function of d_t and θ_t provided that d_i and θ_i are known. So we need at least three sensors to find the unique target location on a plane.

Under the assumption that the target line is far longer than the between-sensor distance, which we will call *Long-range assumption*, we can obtain an approximation by the Taylor series approximation to r_i as

$$r_i \approx d_i\cos(\theta_i - \theta_t) - \frac{d_i^2}{d_t}\sin^2(\theta_i - \theta_t)$$

or

$$r_i \approx d_i\cos(\theta_i - \theta_t).$$

The latter approximation is with the error in the order of d_i^2/d_t. In the rest of this paper, we will consider a 3D localization problem under the Long-range assumption.

3. Probability model

Suppose $n+1$ sensors, sensors $0, 1, \ldots, n$, are used for target localization. Let Z_i, $i = 0, \ldots, n$ denote the product of the speed of light and the time point at which the sensor i received a signal emitted from the target. This value represents the measurement of the range d_{it}, but there is an unknown shift c because the sensor clock cannot be synchronized with the target's signal emission. Let $Y_i = Z_0 - Z_i$, $i = 1, \ldots, n$ be the differences in the range measurements with respect to sensor 0. Assuming that the only source of error is the measurement error, we can write the joint probability density function (pdf) of the measurements as

$$f(z_0, \ldots, z_n | t,\ c) = f(z_0 - c - d_{0t}, \ldots, z_n - c - d_{nt})$$

where the location of the target t and the measurement time shift c are considered as parameters. As a precaution, the symbol t is just an initial of 'target'. The location of the target is denoted by the bold-face $\mathbf{t}$ which is a vector of the three coordinates, d_t, ϕ_t, and θ_t.

It is reasonable to assume that the measurement error made by each sensor is independent of each other:

$$f(z_0, \ldots, z_n | t,\ c) = f_0(z_0 - c - d_{0t}) \cdots f_n(z_n - c - d_{nt})$$

where f_i is the pdf for the measurement error of the i-th sensor. As for the measurement error in the long-distance problem with RF seekers, the thermal noise from the electric circuits of the seeker is possibly a main source of the error. It is known that the thermal noise is well explained by a Gaussian distribution, and the distribution does a good job in practice for long-distance problems. In this regard, we will assume that the measurement errors of the sensors follow a normal distribution with zero mean and covariance matrix $\Sigma = diag(\sigma_0^2, \ldots, \sigma_n^2)$. The joint pdf then equals

$$f(z_0, \ldots, z_n | t,\ c) = (2\pi)^{-(n+1)/2}|\Sigma|^{-1/2}e^{-[(z_0 - c - d_{0t})^2/2\sigma_1^2 + \cdots + (z_n - c - d_{nt})^2/2\sigma_n^2]}.$$

If we let A be the $n \times (n+1)$ matrix given by

$$A = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & \cdots & -1 \end{pmatrix}, \tag{1}$$

then the range difference vector $\mathbf{Y}$ is obtained as

$$\mathbf{Y} = \begin{pmatrix} Y_1 \\ \vdots \\ Y_n \end{pmatrix} = \begin{pmatrix} Z_0 - Z_1 \\ \vdots \\ Z_0 - Z_n \end{pmatrix} = A\mathbf{Z}. \tag{2}$$

Thus the distribution of $\mathbf{Y}$ is $N(\mathbf{r}(t), A\Sigma A')$ where