Signal Processing 133 (2017) 250–259

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Optimal time–frequency distributions using a novel signal adaptive method for automatic component detection

Isabella Reinhold*, Maria Sandsten

Mathematical Statistics, Centre for Mathematical Sciences, Lund University, Sweden

A R T I C L E I N F O

Keywords:
Time–frequency
Multi-component signal
Detection
Performance measure
Heart rate variability

A B S T R A C T

Finding objective methods for assessing the performance of time–frequency distributions (TFD) of measured multi-component signals is not trivial. An optimal TFD should have well resolved signal components (auto-terms) and well suppressed cross-terms. This paper presents a novel signal adaptive method, which is shown to have better performance than the existing method, of automatically detecting the signal components for TFD time instants of two-component signals. The method can be used together with a performance measure to receive automatic and objective performance measures for different TFDs, which allows for an optimal TFD to be obtained. The new method is especially useful for signals including auto-terms of unequal amplitudes and non-linear frequency modulation. The method is evaluated and compared to the existing method, for finding the optimal parameters of the modified B-distribution. The performance is also shown for an example set of Heart Rate Variability (HRV) signals.

1. Introduction

There are many types of non-stationary signals, most of which are multi-component. These signals need to be visualised in time and frequency simultaneously to characterise their time-varying nature. To do this the distribution of the signal energy over the time-frequency plane, i.e. the time-frequency distribution (TFD), can be studied.

The Wigner–Ville distribution (WVD) is a common TFD. For mono-component, linear frequency modulated (FM) signals the WVD gives exactly the instantaneous frequency (IF) making it the optimal TFD for such signals. The problem with the WVD occurs when dealing with multi-component signals or signals disturbed by noise. For such a signal the WVD is not always zero when the signal has no power for a given time–frequency instant. These contributions are called cross-terms and can have twice the amplitude of the signal components. This makes it difficult to distinguish the actual signal components, also called auto-terms, from the cross-terms, [1].

There exist many TFDs which aim to suppress cross-terms by means of filtering the WVD with a kernel, such as Choi–Williams, Zhao–Atlas–Marks [1] and modified B-distribution [2]. However suppression of the cross-terms can also result in loss of resolution of the signal components. Finding good representations of multi-component signals is a complex problem and is still a large field of research [3,4]. When looking at different TFDs for multi-component signals it might be possible to say that some plots look cleaner and thus better. However, assessing the performance based only on this visual comparison is very subjective and finding the optimal parameter for a specific kernel would be very tiresome if not impossible. Not many methods exist, for assessing which TFD is the best for a given signal, especially when dealing with measured signals.

A quantitative performance measure for TFDs of two-component signals, called normalised instantaneous resolution (NIR) performance measure, was presented in [5]. The NIR performance measure makes it possible to compare different TFDs and optimise kernel parameters which control the tradeoff between signal component resolution and cross-term suppression. The NIR performance measure can be used for simulated as well as measured signals and was recently used in [6,7] to find optimal TFDs for different multi-component signals. However, the measure relies on parameters connected to correct detection of the signal components for each time instant of the TFD, and the method used for automatic detection of auto-terms is the one presented in [8]. One restriction of this method is the requirement that the amplitudes of the two signal components are (approximately) equal, which is an assumption that limits the use of the method. The method also fails when signal components are close to each other or has components with non-linear FM law [9], which is a well known restriction of many methods [3]. These restrictions in the detection method narrows the use of the NIR performance measure as the choice of analysed kernel parameters needs to be made with care. This limits the use of the performance measure for automatic optimisation of signal adaptive

* Corresponding author.
E-mail address: isabella@maths.lth.se (I. Reinhold).

http://dx.doi.org/10.1016/j.sigpro.2016.11.028
Received 4 May 2016; Received in revised form 23 November 2016; Accepted 30 November 2016
Available online 02 December 2016

kernels, compared to other methods such as [10]. A large number of methods for identification of signal components exist, e.g. [11–13], who require that cross-terms are well suppressed and locates the maximum peaks as signal components. Other methods such as [14] which uses a method called non-linear squeezing time-frequency transform exist as well. However, these methods require already optimised or semi-optimised TFDs or are more complex and computationally heavy.

This paper presents a new signal adaptive method for automatically detecting signal components in two-component signals which outperforms the method in [8]. The new method is not limited by requiring that the signal components have equal amplitudes. Additionally, the method succeeds in detecting components with non-linear FM laws. Further, the new algorithm overcomes one of the main drawbacks when the estimated parameters are used in the NIR performance measure, as it successfully identifies auto-terms for a larger interval of kernel parameters, allowing for a more objective kernel optimisation. It is also feasible that for two-component signals the new automatic detection method can be used to find the direction of the auto-terms which is used to create an adaptive directional kernel [15,16]. This kernel smooths at each point in the time-frequency domain based on the direction of the energy distribution of the signal.

To illustrate the use of the new signal adaptive automatic detection method together with the NIR performance measure, this paper shows how the optimal kernel parameters for the modified B-distribution [2] of an example set of Heart Rate Variability (HRV) signals with a non-linear component can be obtained. HRV, which is the variation of inter-heartbeat intervals, is measured non-invasively using ECG. It provides information on the autonomic regulation of the cardiovascular system. This means that the HRV is a sensitive indicator of compromised health [17,18]. The HRV has a non-stationary nature, however only recently methods which do not assume stationarity have been evaluated for HRV [19,20]. It is common to study HRV during treadmill running [21,22], making the need for methods of studying HRV in time and frequency concurrently even more important.

The paper is organised as follows. Section 2 provides an introduction to the basics of time-frequency analysis. Section 3 shortly presents the NIR performance measure which will be used and details the new signal adaptive method for automatic detection of the signal components. In Section 4 the performance of the new automatic detection method is evaluated and compared to the performance of the method in [8]. The basis for the evaluation is simulated signals and an example set of HRV signals. The optimal modified B-distributions of the example HRV signals are presented in Section 5. Sections 6 and 7 finish the paper with discussion and conclusions.

2. Time-frequency methods

The Wigner–Ville distribution (WVD),

$$W_z(t, f) = \int_{-\infty}^{\infty} z\left(t + \frac{\tau}{2}\right) z^*\left(t - \frac{\tau}{2}\right) e^{-i2\pi f\tau} d\tau, \tag{1}$$

where * represents the complex conjugate, is a TFD defined using an analytic signal, $z(t)$. The analytic signal is defined such that $Z(f) = 0$ if $f < 0$, where $Z(f) = \mathcal{F}\{z(t)\}$, is the Fourier transform of the signal. The quadratic class of TFDs, a subclass of TFDs where the signal kernel is of quadratic form, can be written as

$$\rho_z(t, f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(t - u, \tau) z\left(u + \frac{\tau}{2}\right) z^*\left(u - \frac{\tau}{2}\right) e^{-i2\pi f\tau} du d\tau, \tag{2}$$

where the time-lag kernel $G(t, \tau)$ is specific for each different quadratic TFD. The convolution of the kernel in (2) is (in most cases) equal to a 2D filtering of the TFD and is used to suppress cross-terms. The design of the kernels is usually done in the ambiguity (doppler-lag) domain, where auto- and cross-terms are more easily differentiable [1].

2.1. Separable and lag-independent kernels

One simple, yet useful, class of kernels is the separable kernels. With the separable kernel the TFD can be written

$$\rho_z(t, f) = g_1(t) \underset{t}{*} W_z(t, f) \underset{f}{*} G_2(f). \tag{3}$$

The convolutions in time and frequency can now be made in either order which simplifies the calculations. It also means that the design of the kernel will be greatly simplified, the 2D filtering operation is replaced by two consecutive 1D filtering operations. A special case of the separable kernel is the lag-independent (LID) kernel. It is obtained by setting

$$G_2(f) = \delta(f), \tag{4}$$

which means that the kernel only will depend on time t. The calculations for the TFD then only require one convolution, in the time direction only

$$\rho_z(t, f) = g_1(t) \underset{t}{*} W_z(t, f). \tag{5}$$

Since the LID kernel only applies one 1D filtering, the resulting TFD will be smoothed in the time direction only. This property makes the LID kernel suitable for slowly varying frequency modulated signals or other signals where cross-terms exist mainly some frequency distance from the auto-terms and single auto-terms do not vary much in frequency. LID-TFDs have been shown to have better performance in characterising HRV signals, compared to other time-frequency methods [23]. The LID kernel can have different distributions, one is the modified B-distribution (MBD), which has been shown to be suitable for HRV signals [24]. The MBD kernel is defined as

$$g_{\mathrm{MBD}}(t) = \frac{\cosh^{-2\beta}(t)}{\int_{-\infty}^{\infty} \cosh^{-2\beta}(\xi) d\xi}, \tag{6}$$

where β is the scaling parameter which determines the trade-off between resolution of signal components and cross-term suppression. The MBD, designed specifically for multi-component IF estimation, is almost cross-term free and has high resolution of signal components in the time-frequency plane [2].

3. Performance measure and a novel signal adaptive method for automatic detection of auto-terms

The NIR performance measure, which combines the concepts of high energy concentration around the IF laws and clearly resolved signal components is presented in [5]. The measure does not take into account some properties usually demanded for TFDs, which impose strict constraints on the TFD design, such as satisfying the marginals [1]. Instead it focuses on resolution of signal components and suppression of cross-terms and sidelobes, which are important for practical use. The measure is defined as

$$P(t) = 1 - \frac{1}{3}\left(\frac{A_S(t)}{A_M(t)} + \frac{1}{2}\frac{A_X(t)}{A_M(t)} + (1 - D(t))\right), \quad 0 \le P(t) \le 1, \tag{7}$$

where $A_S(t)$ is the average absolute amplitude of the largest sidelobes, $A_M(t)$ the average amplitude of the auto-terms (mainlobes), $A_X(t)$ the absolute amplitude of the cross-term and $D(t)$ a measure of the separation of the signal components' mainlobes. It is defined as

$$D(t) = \frac{\left(f_2(t) - \frac{V_2(t)}{2}\right) - \left(f_1(t) - \frac{V_1(t)}{2}\right)}{f_2(t) - f_1(t)}, \tag{8}$$

where $f_1(t)$ and $f_2(t)$ are the centres of the mainlobes and $V_1(t)$ and $V_2(t)$ are the instantaneous bandwidths of the auto-terms, calculated at $\sqrt{2}/2$ of the height of the mainlobe.

For this measure a value close to 1 is a good performance. The