Signal Processing 133 (2017) 270–277

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Asymptotically efficient GNSS trilateration

François Vincent[a,*], Eric Chaumette[a], Christophe Charbonnieras[b], Jonathan Israel[b], Marion Aubault[c], Franck Barbiero[c]

[a] University of Toulouse-ISAE, Department of Electronics, Optronics and Signal, 10 Avenue Edouard Belin, 31055, Toulouse, France
[b] ONERA, The French Aerospace Lab, DEMR/RCP, 2 avenue Edouard Belin, 31055 Toulouse CEDEX 4, France
[c] CNES, The French Space Agency, 18, av. Edouard Belin 31401 TOULOUSE Cedex 9, France

ARTICLE INFO

Keywords:
Approximate Maximum Likelihood estimation
GNSS
GPS
Direct Position Estimation
EXIP
Trilateration

ABSTRACT

Localization based on the reception of radio-frequency waveforms is a crucial problem in many civilian or military applications. It is also the main objective of all Global Navigation Satellite System (GNSS). Given delayed and Doppler shifted replicas of the satellites transmitted signals, the most widespread approach consists in a suboptimal two-step procedure. First, estimate the delays and Dopplers from each satellite independently, then estimate the user position and speed thanks to a Least Square (LS) minimization. More accurate and robust techniques, such as a direct Maximum Likelihood (ML) maximization, that exploit the links in between the different channels exist but suffer from an heavy computational burden that prevent their use in real time applications. Two-steps procedures with an appropriate Weighted LS (WLS) minimization are shown to be asymptotically equivalent to the ML procedure. In this paper, we develop a closed-form expression of this WLS asymptotically efficient solution. We show that this simple expression is the sum of two terms. The first one, depending on the pseudo-ranges is the widespread used WLS solution. The second one is a Doppler-aided corrective term that should be taken into account to improve the position estimation when the observation time increases.

1. Introduction

The main objective of any Global Navigation Satellite System (GNSS) receiver is to estimate its position and speed from all in-view satellite signals. The common approach consists in a two step procedure. First, from each satellite signal, a propagation delay and a Doppler shift are calculated separately. Then, using these estimates, a trilateration step is performed to compute the receiver position and speed. The first stage is easily conducted by correlation of the received signal with the known orthogonal direct-sequence spread spectrum transmitted by each satellite. Depending on the knowledge available on the actual delay and Doppler, a complete (acquisition) or a local search (tracking) is conducted. The second step is a non-linear least-square minimization problem usually performed iteratively by linearising the cost function next to an initial guess. This guess can be computed using a Bancroft algorithm [1] for instance.

This widely used two-step procedure exhibits near optimal performances in open-sky environments but can dramatically degrade in case of complicated scenarios comprising jamming, multipath and channel fading [2]. These effects are known to be the more damageable for the localization precision as they are difficult to compensate by additional aids (such as differential measurements or assisted error modelling). In this case, a direct Maximum Likelihood (ML) position and speed estimation - computed from all in-view satellite signals - will outperform the two-step procedure. Indeed, this so-called Direct Position Estimation (DPE) exploits the fact that the ranges from the satellites are all calculated from the same point, the receiver position, and share a mutual information. The two-step scheme does not exploit this prior knowledge leading to possible incompatible delay estimations and consequently a wrong position calculation. Moreover the information provided by the Doppler shift is rarely exploited in the two-step procedure whereas it handles a useful knowledge on the user position. This piece of information could be very useful to improve the position estimation. It can be noticed that some former systems only exploited this Doppler information for positioning, just like the first global satellite navigation system: TRANSIT. The DPE, that intrinsically exploits this knowledge is known to be asymptotically efficient and unbiased. That is to say, the position and speed correlation error matrix tends toward the lower error bound given by the inverse of the Fisher Information Matrix (FIM). In the GNSS context, the FIM has been

* Corresponding author.
E-mail addresses: francois.vincent@isae.fr (F. Vincent), eric.chaumette@isae.fr (E. Chaumette), Christophe.Charbonnieras@onera.fr (C. Charbonnieras), Jonathan.Israel@onera.fr (J. Israel), Marion.Aubault@cnes.fr (M. Aubault), franck.barbiero@cnes.fr (F. Barbiero).

http://dx.doi.org/10.1016/j.sigpro.2016.11.027
Received 20 July 2016; Received in revised form 3 November 2016; Accepted 29 November 2016
Available online 01 December 2016

calculated in [3] both for the DPE and the two-step approach. Thus the asymptotic precision gain using a DPE procedure instead of the conventional approach can be easily evaluated. The gain can reach one order of magnitude on the position precision in case of large satellite signal power differences or in multipath environments [3]. The DPE has been presented in [4] and extended to the case of an array antenna receiver in [5]. Directly considering position and speed through a single step procedure also allows to introduce any prior information in a natural framework as we work directly with the user position. It can be easily associated with an Inertial Measurement Unit (IMU) for instance. Hence, the DPE philosophy has also been extended to the Bayesian approach in [6,7].

Unfortunately, the price to be paid in using this optimal direct processing is an extensive computational cost that prevents its use in practice. Indeed, DPE requires solving a non-linear multidimensional optimization problem. Possible solutions can be followed to circumvent this problem. For instance, one solution consists in splitting the multidimensional optimization procedure in a number of recursive and simpler searches. The Expectation Maximization (EM) principle, initially introduced in the radio-communication community [8] is an example of such a technique that has been successfully used in the GNSS context. Its extension to antenna array receivers through the Space Alternating Generalized Expectation Maximization (SAGE) is thereby a solution of choice to mitigate multipath [9,10]. Nevertheless, this kind of methods still remains computationally heavy and difficult to be considered in real-time applications.

It has been shown that the two-step procedure cannot outperform the DPE, but when choosing an appropriate weighted matrix for the WLS second step, the two solutions are asymptotically equivalent. The relevant weighted matrix corresponds to the Fisher Information Matrix (FIM) of the model at hand, calculated for the pseudo-ranges and Doppler frequencies. The explanation is linked to the so-called EXtended Invariance Principle (EXIP). EXIP was first introduced in [11] and is based on a re-parametrisation of the problem at hand leading to a less accurate formulation and a simpler solution. Then, these intermediate estimates can be refined to achieve asymptotically the performance of the initial model using an appropriate Weighted Least Square (WLS) minimization. This technique has been used in radar array processing [12,13] for instance. The main idea behind EXIP is to find a reparametrisation that simplify the ML criterion to be maximized. The efficiency property of the ML is maintained (at least asymptotically) during the refining WLS step by using a matched weighting matrix. In the GNSS context, Antreich et al. have exploited the EXIP principle to simplify the joined Direction of Arrival (DoA), delay and Doppler estimation problem in relaxing the constraints on the steering vector structure of the array antenna [14]. In case of a single antenna receiver, it seems natural to re-parametrise the position and speed estimation issue into a delay and Doppler estimation problem assuming an independent processing for each satellite. Then, this approach gives a theoretical grounding to find an asymptotically efficient position and speed estimation thanks to the appropriate EXIP WLS procedure. This approach has been proposed by Closas et al. [2] in the GNSS context, and by Amar and Weiss for the reciprocal problem of active transmitter localization from a set of receivers [15–18]. In all these papers, EXIP has mostly been used to compare the classical two-step procedure with the DPE concept but, to the best of our knowledge, no closed-from formulation of this EXIP DPE solution has been proposed.

In this paper, we propose to calculate this asymptotically efficient solution based on EXIP. The first step of the procedure remains the same as the classical delay and Doppler estimation and can be conducted both by an acquisition procedure or tracking loops. Hence, the classical GNSS receiver architecture does not have to be modified and any advanced tracking processing (Narrow Correlator [19], MEDLL [20],...) can be kept to improve this first stage. Then, we show that we can obtain a closed-form formulation of the FIM leading to a simple WLS solution. Moreover, we simplify this solution to show that it can be seen as the sum of two terms. The first one that only depends on the pseudo-ranges, is the widespread used WLS solution. The second one is Doppler based corrective term. This second term balance, in an optimal way, the information provided by the Doppler to improve the user position. This Doppler correction is negligible when the observation time is short, but can provide useful information if not. This dependence has been exploited in the literature, for instance by Li et al. in [21] and we give here, a theoretical framework to the Doppler-aided positioning estimations.

The paper is organized as follows. Section 2 introduces the framework at hand and defines the signal model to be used. Then we make use of the EXIP to compute the optimal weight matrix in Section 3. Then, we simplify this solution in Section 4 to give an insightful interpretation. Numerical illustrations are provided in Sections 5 and 6 draws conclusions.

2. Data model

We assume that K scaled, delayed and Doppler-shifted front waves, transmitted by each in-view satellite impinge on a GNSS receiver antenna. Under the narrowband assumption, the complex baseband model can be written as follows:

$$x(t) = \sum_{k=0}^{K-1} \alpha_k.\ c_k(t-\tau_k).\ e^{2i\pi f_k t} + n(t) \tag{1}$$

where

- α_k denotes each complex satellite signal amplitude, supposed to be deterministic and unknown,
- $c_k(t)$ stands for the transmitted complex baseband navigation signal spread by the pseudo-random code corresponding to the k-th satellite,
- f_0 is the carrier frequency,
- $n(t)$ corresponds to an additive zero-mean white Gaussian noise with variance σ^2,
- and τ_k, f_k are respectively the delay and Doppler frequency shift of the k-th satellite signal, observed from the receiver.

We suppose that N snapshots are sampled at a $F_s = \frac{1}{T_s}$ rate from $x(t)$, so that we can write:

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{\alpha} + \boldsymbol{n} \tag{2}$$

where

- $\boldsymbol{x} = [x(0)\ldots x((N-1)T_s)]^T$,
- $\boldsymbol{A} = [\boldsymbol{a}_0\ldots\boldsymbol{a}_{K-1}]$ is the manifold corresponding to all in-view satellite signals, with $\boldsymbol{a}_k = [c_k(-\tau_k)\ldots c_k((N-1)T_s-\tau_k).\ e^{-2i\pi f_0 b_k(N-1)T_s}]^T$,
- $\boldsymbol{\alpha} = [\alpha_0\ldots\alpha_{K-1}]^T$ and,
- $\boldsymbol{n} = [n(0)\ldots n((N-1)T_s)]^T$

with $.^T$ being the transpose operation.

The observed delay τ_k and delay drift $b_k = -\frac{f_k}{f_0}$ depends on the actual relative distance and velocity from the satellite k to the receiver, as well as secondary propagation effects (atmospheric and ionospheric additional delays,...) and receiver or transmitter defaults (clock bias and drift). They can be expressed as follows:

$$\tau_k \simeq \frac{\|\boldsymbol{p}_k - \boldsymbol{p}\|}{c} + \tau_0 + \delta\tau_k b_k \simeq \frac{(\boldsymbol{v}_k - \boldsymbol{v})^T.\ \boldsymbol{u}_k}{c} + b_0 + \delta b_k \tag{3}$$

where:

- $\boldsymbol{p}, \boldsymbol{v}, \boldsymbol{p}_k$ and $\boldsymbol{v}_k \in \Re^3$ are respectively the position and velocity vectors of the receiver and of the k-th satellite,