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Nodally integrated implicit gradient reproducing kernel particle method for convection dominated problems

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Highlights

- A general framework for SU/PG, G/LS, and SGS stabilization methods.
- Equivalence between reproducing kernel implicit gradient and diffuse derivative.
- Stabilized lower order quadrature scheme for convection dominated problems.

Abstract

Convective transport terms in Eulerian conservation laws lead to numerical instability in the solution of Bubnov–Galerkin methods for these non-self-adjoint PDEs. Stabilized Petrov–Galerkin methods overcome this difficulty, however gradient terms are required to construct the test functions, which are typically expensive for meshfree methods. In this work, the implicit gradient reproducing kernel particle method is introduced which avoids explicit differentiation of test functions. Stabilization is accomplished by including gradient terms in the reproducing condition of the reproducing kernel approximation. The proposed method is computationally efficient and simplifies stabilization procedures. It is also shown that the implicit gradient resembles the diffuse derivative originally introduced in the diffuse element method in Nayroles et al. (1992), and maintains the desirable properties of the full derivative. Since careful attention must be paid to efficiency of domain integration in meshfree methods, nodal integration is examined for this class of problems, and a nodal integration method with enhanced accuracy and stability is introduced. Numerical examples are provided to show the effectiveness of the proposed method for both steady and transient problems.

Keywords: Convection dominated problems; Reproducing kernel particle method; Implicit gradient; Stabilization; Nodal integration

1. Introduction

It is well known that the application of standard Bubnov–Galerkin methods to convection dominated problems yields oscillatory solutions. Convective transport terms in Eulerian conservation laws lead to numerical instability in the solution of these problems, and the instability manifests when fine-scale features such as boundary layers

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are present in the solution. Stabilized Petrov–Galerkin methods [1–3] have been developed which provide superior stability over standard Bubnov–Galerkin methods. Alternatively, methods that either directly or indirectly include fine-scale features such as the variational multiscale method [4,5], residual free bubbles [6], or subgrid scale (SGS) methods [7], have been introduced to resolve the issue with using Galerkin methods for this class of problems. These two approaches are intimately related, and the motivation and interpretation of each is not necessarily mutually exclusive (cf. [4,5,7–9]).

Stabilized methods predate both bubble methods and variational multiscale methods, although the relationship between the three was later shown in [4]. In these methods, portions of the differential operator are included in the test function, which can be shown to ensure stability of the Galerkin solution with the proper selection of a stabilization parameter. The streamline upwind Petrov–Galerkin (SU/PG) method first presented in [1] gave stabilization in a consistent manner, and put to rest notions of artificial diffusion. Thereafter, an analysis of SU/PG was given in [10] and the Galerkin/least squares (G/LS) method [2] was introduced to give a method grounded in stability and convergence. The SGS stabilized finite element method [11,3] reveals that static condensation of bubble functions is equivalent to the use of the negative adjoint operator for a stabilized method. The stability and convergence of the stabilized finite element methods have been well investigated [10,2,3].

The difficulties with convection dominated problems and methods used to address them discussed above are largely applicable to meshfree methods. In addition, due to the unique properties of these discretizations, several novel approaches have been taken to address the issue apart from traditional stabilization. Early on, the multi-resolution reproducing kernel particle method (RKPM) [12] was introduced to resolve internal layers in the advection–diffusion equation. The finite point method [13,14] employed characteristics to obtain a stable solution. The meshless local Petrov–Galerkin method has also been developed for convection dominated problems in [15,16], with upwinding schemes for the trial functions and local sub-domains. Several more traditional approaches have also been proposed for meshfree methods. Stabilized RKPM [17] has been applied to flow problems, and suitable stabilization parameters have been discussed in [18,19]. A higher order accurate time integration scheme [20] has been proposed for meshfree methods for convection dominated problems. Recently, the variational multiscale framework has been applied to RKPM for convection dominated problems and other related problems in [21–24], using discrete representations of the fine scales.

An ever-present issue in Galerkin meshfree methods is domain integration. Due to the rational nature of shape functions, and often misaligned supports and integration cells, computationally demanding high order integration is often required for solution accuracy [25]. One approach that has been taken is to impose exactness in the Galerkin solution with quadrature. A stabilized conforming nodal integration (SCNI) was proposed in [26,27], which constructs smoothed gradients such that the linear patch test is satisfied, attaining optimal convergence for linear basis. More recently, this concept has been cast under a framework of variational consistency conditions in [28], where it was shown that by constructing gradients such that higher order patch tests are satisfied, optimal convergence rates associated with the order of the approximation space can be attained. Adding additional stress points in a variationally consistent manner has also been proposed to either facilitate higher order accuracy when additional enrichment functions are present [29], or increase the stability of variationally consistent methods [30,31]. In both cases, maintaining variational consistency has been the key to their overall effectiveness.

Implicit gradients have been introduced for meshfree methods for various purposes. The basic idea is to embed the desirable properties of the derivatives in the approximation, e.g., partition of nullity, but do so without differentiating the shape functions. The first use of implicit gradients was for synchronized convergence in [32,33], where it was shown that the approach could also be used as a way to avoid taking derivatives and thus attain efficiency. Implicit gradients have also been used for regularization in strain localization problems [34] to avoid the need of ambiguous boundary conditions associated with the standard gradient-type regularization methods. In [35], they were utilized for easing the computational cost of meshfree collocation methods, which require higher order derivatives. In this paper, the implicit gradient reproducing kernel particle method (IG-RKPM) is introduced for convection dominated problems. A gradient reproducing condition is employed which allows stabilization under a unified framework without the explicit construction of costly gradient terms. The technique is thus computationally efficient, simplifies stabilization procedures considerably, and it requires only a small modification to the standard shape RKPM function. Since the choice of domain integration is critical for meshfree methods to be effective, SCNI is investigated for convection dominated problems. It is shown that while stable when solving self-adjoint equations, it can yield unstable solutions for this class of problems, exhibiting the instability it was originally designed to preclude. As such, high order SCNI