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Review

Relevance of polynomial matrix decompositions to broadband blind signal separation

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ABSTRACT

The polynomial matrix EVD (PEVD) is an extension of the conventional eigenvalue decomposition (EVD) to polynomial matrices. The purpose of this article is to provide a review of the theoretical foundations of the PEVD and to highlight practical applications in the area of broadband blind source separation (BSS). Based on basic definitions of polynomial matrix terminology such as parahermitian and paraunitary matrices, strong decorrelation and spectral majorisation, the PEVD and its theoretical foundations will be briefly outlined. The paper then focuses on the applicability of the PEVD and broadband subspace techniques — enabled by the diagonalisation and spectral majorisation capabilities of PEVD algorithms — to define broadband BSS solutions that generalise well-known narrowband techniques based on the EVD. This is achieved through the analysis of new results from three exemplar broadband BSS applications — underwater acoustics, radar clutter suppression, and domain-weighted broadband beamforming — and their comparison with classical broadband methods.

1. Introduction

Over the last decade, algorithms that extend the eigenvalue decomposition (EVD) to the realm of polynomial matrices have had a growing impact on signal processing theory and practice, mainly because they can be used to solve generalisations of narrowband problems typically addressed by the EVD, including subspace decomposition. The extension of EVD to parahermitian (PH) polynomial matrices, referred to as polynomial matrix EVD (PEVD), gives an immediate broadband generalisation of the concepts of signal and noise subspaces, and hence subspace decompositions. Just as principal component analysis (PCA) based on the EVD is fundamental to most narrowband BSS formulations, the PEVD can be a powerful tool for broadband or convolutive blind source separation (BSS).

The classical approach to narrowband BSS begins by exploiting second-order statistics to generate uncorrelated sequences from narrowband, instantaneously mixed signals by performing principal component analysis (PCA) [1,2]. PCA is usually obtained through matrix factorisation by means of a unitary matrix decomposition, such as the singular value (SVD) or eigenvalue decomposition (EVD) [3,4]. To complete the BSS process, a "hidden" rotation matrix is determined via on higher-order statistics (HOS), which permutes entries to achieve spectral coherence across frequency bins. With little or no prior knowledge and minimal assumptions, a BSS method can often be used to extract a wanted signal from among interference signals. However, the wanted signal is in no way accentuated by these underlying assumptions.

Incorporation of a priori knowledge of the signals into the BSS problem can be formulated in the framework of signal decompositions and matrix factorisations, and address statistical dependence, periodicity, spectral shape, time coherence or smoothness [5–8]. The goal often is to estimate a reduced coordinate space, which provides a more accurate physical representation of the sources or mixing parameters.

The above signal decompositions are based on an instantaneous mixing model, where the propagation of signals from sources to the array is modelled as a scalar mixing matrix. However, in many important applications such as broadband array processing, convolutive mixing — or a matrix of finite impulse response (FIR) filters — must be used instead. The transfer function of such a matrix of FIR filters forms a polynomial matrix, which can accurately model effects such as multipath propagation, or the lag-dependent correlation between different broadband sensor signals. SVD- or EVD-based decompositions generally can only decorrelate instantaneously, i.e., only for zero lag. Following convolutive mixing, strong decorrelation

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[9] eliminates correlation for all lag values, and can be achieved using a well-designed matrix of FIR filters.

In the past, broadband BSS has been addressed by performing narrowband BSS at each frequency bin simultaneously, through application of the discrete Fourier transform (DFT) — commonly referred to as independent frequency bin (IFB) processing. However, coherence restoration is required after BSS via permutation matrices applied in every bin [10,11]. An alternative is to adopt coherent signal subspace-related methods, which generally require some prior knowledge of signals, such as direction and fractional bandwidth, to coherently combine covariance matrices across different bins in order to create an approximately narrowband problem [12–14].

The formulation and decomposition of polynomial matrices presents an alternative to these classical broadband BSS approaches. Polynomial matrices have been used for many years, e.g., in the area of control [15] or broadband subspace decomposition and adaptive sensor arrays [16–18]. Various polynomial matrix factorisations have been addressed, such as the Smith–Macmillan form [19], or polynomial matrix factors that are paraunitary (PU) or lossless [20–33]. Typically, the filter is chosen to optimise a specific objective function for a known input power spectral density (PSD), such as coding gain [9,20,23,31,32] for subband coding.

The space–time covariance matrix derived from broadband sensor data includes auto- and cross-correlation terms, whose symmetries create the specific form of a parahermitian (PH) polynomial matrix. The PEVD of such a PH matrix was proposed in [16,25,26], and leads to a factorisation where a diagonal PH matrix containing the polynomial eigenvalues is pre- and post-multiplied by a PU matrix, or lossless, filter bank. The existence of such a factorisation based on FIR PU matrices is not ascertained [19], but suggested that it exists at least in good approximation [34].

The polynomial eigenvalues of a PEVD represent the power spectral densities of the strongly decorrelated signals. Depending on the PEVD algorithm (discussed below), the eigenvalues can be ordered akin to the singular values of the SVD at every frequency. The ordering of the spectra in this way is called spectral majorisation [9,26], and is useful in a number of applications.

An initial iterative scheme to approximate the PEVD, the second order sequential best rotation (SBR2) algorithm [26], has triggered similar or related efforts [28–33,35–40]. SBR2 has been proven to converge [26,31], and found to approximate the ideal PEVD very closely [34]. A coding-gain based version of SBR2 (SBR2C) was shown to offer improved convergence in [31].

More recently, the sequential matrix decomposition (SMD) and maximum-element (ME-SMD) algorithms [33] have shown superior convergence due to their advanced energy transfer ability, as compared to other iterative algorithms. The multiple-shift variant of the ME-SMD in [36] has shown marked improvement in convergence speed compared to SMD.

The SMD and SBR2 algorithms have been successfully applied to a number of broadband extensions of narrowband problems, traditionally addressed by the EVD, including, e.g., broadband array processing [41–47], channel coding [48], broadband communications [49], spectral factorisation [50], convolutive BSS [42,46], and the design of FIR PU filter banks for subband coding [31,32]. The recent parallelisation of SBR2 in [35], for field programmable gate arrays, has enabled application of SBR2 to real-time problems using embedded processing [51].

The advantage of polynomial matrix decompositions over IFB processing lies in the natural ability of broadband decomposition algorithms to preserve and exploit the coherence of signals.

As a particular example of applying the PEVD to convolutive BSS with prior knowledge, in [42] a broadband extension to the narrowband semi-blind signal approach in [52] has been performed. The broadband equivalent method used some prior information about the direction of sources acquired by a broadband array was embedded to achieve an enhanced separation of sources. This can be combined with other broadband approaches, such as polynomial MUSIC [44,45], to estimate the prior knowledge that can then be passed to the BSS problem.

The aim of this paper is twofold: (i) provide an overview of polynomial matrix factorisation and (ii) discuss applications in the area of broadband BSS. In Section 2, the PEVD and related fundamental concepts, such as paraunitarity, strong decorrelation and spectral majorisation, are introduced. In Sections 3 and 4, we present solutions and new results to three important problems via a PEVD-based broadband beamformer and domain-weighted PEVD. The results are compared to classical methods, which contrast the natural ability of broadband subspace decomposition algorithms to preserve and exploit the coherence of signals. Lastly, conclusions are drawn in Section 5.

2. Polynomial matrix eigenvalue decomposition

2.1. Notation

In this paper matrices and vectors are represented by bold uppercase and bold lowercase characters, e.g., $\mathbf{X}$ and $\mathbf{x}$, respectively. An element of $\mathbf{X}$ is denoted by x_{jk}. Complex conjugation, matrix transposition and Hermitian transposition are indicated by the superscripts *, T and H, respectively. A $p \times p$ (complex-valued) Hermitian matrix $\mathbf{R} \in \mathbb{C}^{p \times p}$ has the property $\mathbf{R} = \mathbf{R}^{\mathrm{H}}$; a unitary matrix $\mathbf{U} \in \mathbb{C}^{p \times p}$ has the property $\mathbf{U}^{\mathrm{H}}\mathbf{U} = \mathbf{U}\mathbf{U}^{\mathrm{H}} = \mathbf{I}_p$, where $\mathbf{I}_p$ is the $p \times p$ identity matrix.

Polynomial matrices are polynomials with matrix-valued coefficients, or matrices with polynomial elements [15,19]. An $n \times q$ polynomial matrix in the indeterminate variable z^{-1} is denoted by

$$\boldsymbol{A}(z) = \sum_{\tau=t_1}^{t_2} \mathbf{A}[\tau] z^{-\tau}, \tag{1}$$

where $a_{ij}(z) = \sum_{\tau=t_1}^{t_2} a_{ij}[\tau] z^{-\tau}$, $t_1 \le t_2$, $\tau \in \mathbb{Z}$ and $a_{ij}[\tau] \in \mathbb{C}$, is an element of $\mathbf{A}[\tau]$. Hence, coefficient matrices of $\boldsymbol{A}(z)$ can be written as $\mathbf{A}[t_1], \ldots, \mathbf{A}[t_2]$; e.g., the coefficient matrix of lag zero (lag-zero coefficient matrix) is denoted $\mathbf{A}[0]$. Note that the effective order of $\boldsymbol{A}(z)$ is $t_2 - t_1$. A transform pair as in (1) is abbreviated as $\boldsymbol{A}(z) \bullet\!\!-\!\!\circ\, \mathbf{A}[\tau]$. Also note that parentheses express dependency on continuous variables, while square brackets denote dependency on discrete variables.

2.2. Space–time covariance matrix

It is well-known that instantaneous spatial correlation, i.e., correlation between pairs of signals sampled at the same instant in time, can be removed using the EVD and SVD [4]. Therefore, the SVD (or EVD) can be used to decorrelate instantaneous mixtures, e.g., for the case of narrowband sensor arrays. However, convolutively mixed signals, or signals derived from a broadband sensor array, cannot be decorrelated in this way. The sensor-weight values required to correct for the time delay between sensors are different for different frequencies. Frequency dependent weights can be realised using FIR filters, which form a frequency dependent response for each sensor signal in order to compensate the phase difference for the different frequency components. The sensors thus sample the propagating wave field in both space and time.

Hence, in order to express the signals at the sensors, we modify the well-known instantaneous-mixing (or narrowband) model to take account of this process

$$\mathbf{x}[t] = \mathbf{A}[t] * \mathbf{s}[t] + \boldsymbol{\eta}[t], \tag{2}$$

where the asterisk denotes multi-input multi-output (MIMO) convolution [51], $\mathbf{A}[\tau] \circ\!\!-\!\!\bullet\, \boldsymbol{A}(z)$ is the $p \times q$ mixing matrix of FIR filters $a_{ij}[t]$ and $\mathbf{s}[t] \in \mathbb{C}^q$ and $\boldsymbol{\eta}[t]$ represent independent source and noise signals.

The signals $\mathbf{x}[t]$ will generally be correlated over multiple time lags,

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