



# Mode-dependent filter design for Markov jump systems with sensor nonlinearities in finite frequency domain

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## ABSTRACT

This paper is concerned with the filter design for Markov jump systems with incomplete transition probabilities subject to sensor nonlinearities. Moreover, the frequency of disturbance ranges in a finite interval. To set up a solvable solution to cast the filter parameters, nonlinearities induced by unknown transition probabilities are coped with the transition probability property and the S-procedure is adopted to handle sensor nonlinearities. With these strategies, sufficient conditions for the filtering error systems to be stochastically stable with the required finite frequency performance are established firstly. Then, a finite frequency filter design method is proposed in terms of linear matrix inequalities. The proposed finite frequency filter method covers the full frequency as a special case. Its effectiveness is verified by a numerical example.

## 1. Introduction

Markov jump systems belong to the category of stochastically hybrid systems and have been widely employed to model dynamic systems caused by random abrupt variations. Applications of this kind of system can be found in the area of manufacture systems, economical systems and network control systems. In terms of its theoretical research, fruitful results have been reported in the literature on stability analysis and stabilization, slide mode control, fault detection, adaptive control and so on [1–15]. Since transition probabilities dominate the transition among subsystems, they are presumed to be known in advance.

Recently, much attention has been devoted to the study of Markov jump systems with incomplete transition probabilities. The reason is that the accurate transition probabilities for practical systems may be hard or costly to be acquired [16]. To be consistent with the engineering requirements, uncertain transition probabilities are characterized by the norm-bounded or polytopic structure [17,18]. Consequently, robust control methodologies are convenient to dispose of them. In contrast to uncertain transition probabilities with known structure, they are permitted to be known or unknown in [19]. In this scenario, one main issue is how to linearize nonlinearity induced by unknown transition probabilities. To get less conservative results, the free weighting matrix technique is applied to make full use of known

transition probabilities [20]. Taking into account of known and uncertain transition probabilities,  $H_\infty$  static output feedback control of Markov jump linear uncertain systems is discussed in [21]. Alternatively, in [22], uncertain transition probabilities are approximated by Gaussian probability density.

On another research front line, state estimation or filtering of Markov jump system has primary importance in the field of signal processing and communication [23]. Among various existing approaches, the  $H_\infty$  filtering has been favored by many researchers [24–27]. It is noted that sensor outputs in these results are linear. Unfortunately, nonlinear outputs may be measured due to harsh environments and finite register-length of sensors [28,29]. Collecting these factors, a delay-dependent  $H_\infty$  filtering approach for Markov jump systems with sensor nonlinearities is developed in [28]. An asynchronous  $l_2 - l_\infty$  filter for Markov jumping systems with randomly sensor nonlinearities is addressed in [29]. It is to note that these results are built on the full frequency domain. Unfortunately, the frequency ranges of exogenous disturbances may be known beforehand [30]. Taking a vehicle suspension system as an example, the main control task for this system is to attenuate the vertical vibrations of 4–8 Hz which is sensitive to human body. To ensure the comfortability, it is desired to alleviate the effect of these finite frequency vibrations. If the accessible frequency information is not used, the resulting design method could be conservative [30–33]. Up to now, due to the technical

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difficulty, there is no corresponding filtering result for Markov jump system with incomplete transition probabilities and sensor nonlinearities.

Excited by the above observations, this paper is dedicated to the mode-dependent filter design for Markov jump systems with sensor nonlinearities in finite frequency range. Transition probabilities are known, uncertain with known bounds and unknown, and nonlinear sensor outputs are expressed by the sector bounded form. Effective measurements are exploited to settle sensor nonlinearities and nonlinearities induced by unknown transition probabilities. According to these measurements, a finite frequency method for the filtering error systems to be stochastically stable with the prescribed performance is explored. Based on the proposed approach, an unified filter design framework is constructed in terms of linear matrix inequalities. It is shown that the finite frequency method may be less conservative than that of the full frequency. The validity of the proposed method is demonstrated by a numerical example.

The organization of this article is given as follows. The problem statement and some preliminaries are introduced in Section 2. Three theorems are put in Section 3. A numerical example is given to illustrate the effectiveness of the proposed approach in Section 4. Lastly, Section 5 concludes the paper.

**Notation:** Throughout the paper,  $Y > 0$  ( $< 0$ ) means its positive (negative) definite.  $\mathbb{R}^n$  indicates the  $n$ -dimensional Euclidean space and  $\mathbb{R}^{n \times m}$  is the set of all  $n \times m$  real matrices.  $\mathbb{L}_2$  denotes the square integrable vector functions over  $[0, \infty)$  with norm  $\|x\|_2 = \int_0^\infty x(t)^T x(t) dt$ . The transposed  $M$  is  $M^T$ .  $*$  stands for the entries symmetry. The mathematical expectation is presented by  $\mathbb{E}$ .  $j$  is an imaginary unite.  $(X + X^T)$  is shorten as  $He(X)$ . All matrices, if not explicitly clarification, are assumed to have compatible dimensions.

## 2. Problem statement and preliminaries

Consider a continuous Markov jump system as

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B(r(t))w(t) \\ y(t) = C_1(r(t))x(t) + D_1(r(t))w(t) \\ z(t) = C_2(r(t))x(t) + D_2(r(t))w(t) \\ y_s(t) = \phi(y(t)) \end{cases} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$ ,  $z(t) \in \mathbb{R}^r$ ,  $y_s(t) \in \mathbb{R}^m$  and  $w(t) \in \mathbb{R}^q$  are the state vector, regulated output, sensor output and energy bounded disturbances with limited frequency ranges respectively.  $A(r(t)) \in \mathbb{R}^{n \times n}$ ,  $B(r(t)) \in \mathbb{R}^{n \times q}$ ,  $C_1(r(t)) \in \mathbb{R}^{m \times n}$ ,  $D_1(r(t)) \in \mathbb{R}^{m \times q}$ ,  $C_2(r(t)) \in \mathbb{R}^{r \times n}$  and  $D_2(r(t)) \in \mathbb{R}^{r \times q}$  are system matrices with approximate dimension.  $\phi(\bullet)$  is a vector-valued nonlinear function.  $r(t) (t \geq 0)$  is a continuous Markov processes and takes values in a set  $I = \{1, 2, \dots, N\}$ .

The transition from mode  $i$  to mode  $l$  of  $r(t)$  satisfies

$$Pr(r(t+h) = l | r(t) = i) = \begin{cases} \pi_{il}h + o(h), & \text{if } l \neq i \\ 1 + \pi_{ii}h + o(h), & \text{if } l = i \end{cases}$$

where  $h > 0$  and  $\lim_{h \rightarrow 0} \frac{o(h)}{h} = 0$ ,  $\pi_{il} \geq 0 (i, l \in I, l \neq i)$  and  $\pi_{ii} = -\sum_{l \neq i}^N \pi_{il}$ .

Since it is costly to exactly sample the transitions among different modes, there are inevitable to be incomplete [19,21]. Consequently, transition probabilities are allowed to be known, uncertain and unknown as follows

$$\begin{bmatrix} \pi_{11} & ? & \alpha_{13} & ? \\ ? & ? & ? & ? \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \\ \pi_{41} & ? & ? & \pi_{44} \end{bmatrix} \quad (2)$$

where ? means that the corresponding elements are inaccessible and  $\alpha_{il}$  denotes uncertain with known lower and upper bounds  $(\underline{\alpha}_{il}, \bar{\alpha}_{il})$ . To distinguish the accessibility of all transition probabilities,  $\mathcal{L}_k^i$  and  $\mathcal{L}_{uk}^i$  are defined as follows,

$$\mathcal{L}_k^i = \{\pi_{il} \text{ is known or uncertain, } l \in I\} \quad \mathcal{L}_{uk}^i = \{\pi_{il} \text{ is unknown, } l \in I\} \quad (3)$$

Before proceeding further, an assumption borrowed from [34,35] is given below:

**Assumption 1.** The vector-valued nonlinear function  $\phi(\bullet)$  satisfying the following sector condition

$$(\phi(\vartheta) - K_1\vartheta)^T(\phi(\vartheta) - K_2\vartheta) \leq 0,$$

where  $\vartheta$  is a vector,  $K_1$  and  $K_2$  are diagonal real matrices.

Without loss of generality, it is assumed that  $K_2 > K_1$ . Then  $K = K_2 - K_1$  is a positive-definite matrix. Therefore,  $y(t)$  is decomposed to a linear and a nonlinear parts as below [28,29]

$$y_s(t) = K_1 y(t) + \phi_s(y(t)), \quad (4)$$

with

$$\phi_s^T(y(t))(\phi_s(y(t)) - Ky(t)) \leq 0. \quad (5)$$

As mentioned in the Introduction, all possible frequency ranges are described by the following set [30]

$$\Theta_{\varpi} := \{\varpi \in \mathbb{R} | \tau(\varpi - \varpi_1)(\varpi - \varpi_2) \leq 0\} \quad (6)$$

where  $\tau = \pm 1$ ,  $\varpi_1$  and  $\varpi_2$  are known real scalars, and  $\varpi$  is frequency variable.

**Remark 1.** The proposed finite frequency range covers low, middle and high frequencies by choosing different combinations of  $\tau$  and  $\varpi_1$  and  $\varpi_2$ . To be specific, let  $\varpi_1 = -\varpi_2$  and  $\tau = 1$ , then one has the low frequency conditions. The middle case are attained by selecting  $\tau = 1$  and  $\varpi_1 \neq \varpi_2$ . To get the high case, set  $\varpi_1 = -\varpi_2$  and  $\tau = -1$ .

To ease presentation, if  $r(t) = i$ ,  $A(r(t))$ ,  $B(r(t))$ ,  $C_1(r(t))$ ,  $D_1(r(t))$ ,  $C_2(r(t))$  and  $D_2(r(t))$  are simplified as  $A_i$ ,  $B_i$ ,  $C_{1i}$ ,  $D_{1i}$ ,  $C_{2i}$  and  $D_{2i}$ .

In this paper, the following full order filter is designed

$$\begin{cases} \dot{x}_f(t) = A_{fi}x_f(t) + B_{fi}y(t) \\ z_f(t) = C_{fi}x_f(t) + D_{fi}y(t) \end{cases} \quad (7)$$

where  $x_f(t) \in \mathbb{R}^n$  is filter state,  $z_f(t) \in \mathbb{R}^r$  is estimated output,  $A_{fi}$ ,  $B_{fi}$ ,  $C_{fi}$  and  $D_{fi}$  are filter parameters to be designed.

Combining (1), (4), (5) and (7), the augmented filtering error system is

$$\begin{cases} \dot{\xi}(t) = \bar{A}_i \xi(t) + \bar{B}_{si} \phi_s(y(t)) + \bar{B}_i w(t) \\ e(t) = \bar{C}_i \xi(t) + \bar{D}_{si} \phi_s(y(t)) + \bar{D}_i w(t) \end{cases} \quad (8)$$

where

$$\begin{aligned} \xi(t) &= \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, e(t) = z(t) - z_f(t), \bar{A}_i = \begin{bmatrix} A & 0 \\ B_{fi}K_1C_{1i} & A_{fi} \end{bmatrix}, \bar{B}_{si} = \begin{bmatrix} 0 \\ B_{fi} \end{bmatrix}, \bar{B}_i \\ &= \begin{bmatrix} B_i \\ B_{fi}K_1D_{1i} \end{bmatrix}, \bar{C}_i = [(C_{2i} - D_{fi}K_1C_{1i}) \quad -C_{fi}], \bar{D}_{si} = -D_{fi}, \bar{D}_i \\ &= -D_{fi}K_1D_{1i}. \end{aligned}$$

Prior to addressing the considered filtering problem, inspired by [30,31], the definition on finite frequency performance index  $\gamma$  is given below at first

**Definition 1.** The filtering error system (8) has a finite frequency disturbance attenuation performance  $\gamma$  if, under zero initial condition, the inequality

$$\mathbb{E} \left\{ \int_0^\infty e^T(t) e(t) dt \right\} \leq \gamma^2 \mathbb{E} \left\{ \int_0^\infty w^T(t) w(t) dt \right\}, \quad (9)$$

holds for (8) with  $w(t) \in \mathbb{L}_2$  satisfying

$$\mathbb{E} \left\{ \int_0^\infty \tau(\varpi_1 x(t) + j\dot{x}(t))(\varpi_2 x(t) + j\dot{x}(t))^T dt \right\} \leq 0. \quad (10)$$

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