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Optimal duration-bandwidth localized antisymmetric biorthogonal wavelet filters

Manish Sharma[b,*], Abhinav Dhere[c], Ram Bilas Pachori[b], Vikram M. Gadre[a]

[a] Department of Electrical Engineering, Indian Institute of Technology Bombay, Mumbai, India
[b] Discipline of Electrical Engineering, Indian Institute of Technology Indore, Indore, India
[c] Department of Electronics and Communication Engineering, Acropolis Institute of Technology and Research, Indore, India

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ABSTRACT

We present a design of a new class of compactly supported antisymmetric biorthogonal wavelet filter banks which have the analysis as well as the synthesis filters of even-length. Here, the analysis and the synthesis filters are designed to have minimum joint duration-bandwidth localization (JDBL). The design of filters has been formulated as a direct time-domain linearly constrained eigenvalue problem that does not involve any parametrization and iterations. The optimal analysis and synthesis filters have been obtained as the eigenvectors of the positive definite matrices. The closed form analytic expression for the objective function has been presented. The perfect reconstruction and regularity conditions have been incorporated in the design by employing time-domain matrix characterization. The method can control duration and bandwidth localizations of the analysis and synthesis filters, independently. A few design examples have been presented and compared with previous works. The performance of the optimal filter banks designed by employing the proposed method has been evaluated in image coding and signal denoising applications.

1. Introduction

Wavelet filter banks (FBs) are deployed in many applications of signal processing and communications. The design of wavelet FBs can be reduced to a constrained optimization problem wherein the perfect reconstruction (PR) and regularity constraints are imposed, and a desired objective such as compaction energy, stopband energy, frequency selectivity, roll off factor, ripples and joint duration-bandwidth localization (JDBL) of the filters is optimized [1–3]. The selection of optimality criterion depends upon the application.

The JDBL of filters plays a pivotal role in certain applications of signal processing and communications. Wilson and Granlund [4] observe that JDBL optimized filters perform well in image segmentation, feature extraction, edge detection, and image compression. Monro et al. [5] and Morris et al. [6] demonstrate that JDBL optimized orthogonal wavelet FBs perform better than other FBs in image coding applications. Davidson et al. [7] find that the JDBL optimized FBs outperform other filters in pulse shaping system to mitigate inter symbol interference (ISI). Dandach and Siohan [8] observe an excellent performance of JDBL optimized FBs in reducing ISI and inter-channel interference in orthogonal frequency division multiplexing (OFDM) multicarrier systems. The minimum bandwidth orthogonal wavelet FBs perform well in some applications such as image compression [9] and denoising of electrocardiogram (ECG) signal [10]. Thus, the JDBL is an important attribute in selecting FBs.

It has been observed that in certain applications even-length wavelet FBs perform better than odd-length FBs. Villasenor et al. [11] find that even-length wavelet FBs perform better in image coding applications. Villasenor et al. [11] find that the even-length FBs have more shift-invariance than the odd-length FBs. Kronander [12] finds that the even-length FBs perform better than the odd-length FBs in video coding to reduce flicker noise. Further, in the design of dual-tree complex wavelet transform [13], even-length FBs are required, as the FBs provide half-sample delay. The central distinctive feature between odd and even-length FBs is that the former generates symmetric wavelets while the latter yields antisymmetric wavelets.

Tay [14,15] and Muthuvel and Makur [16] design even-length biorthogonal FBs in which the filters have minimum ripple energies in pass band and stop band. However, the authors do not consider JDBL as an optimality criterion. Recently, the design of JDBL optimized windows has received a lot of attention [17,18]. Morris and his collaborators [6,19,10,20] and Monro et al. [5] design JDBL optimized orthogonal FBs, and evaluate the performance of their FBs in several applications. However, they do not consider the design of JDBL

* Corresponding author.
E-mail addresses: manishsharma.iitb@gmail.com (M. Sharma), abhinavdhere@gmail.com (A. Dhere), pachori@iiti.ac.in (R.B. Pachori), vmgadre@ee.iitb.ac.in (V.M. Gadre).

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optimized biorthogonal FBs. Tay [21] designs a class of JDBL optimized biorthogonal FBs called half band pair filter bank using parametric Bernstein polynomial. However, the filters of this class always have an odd length. Moreover, the design method is highly restrictive. In the previous works, Sharma et al. [22,23] design a general class of odd length biorthogonal JDBL optimized symmetric wavelet FBs where all filters are Type-I linear phase filters. However, the authors do not present a design for antisymmetric biorthogonal wavelet FBs (ABWFBs). In this work, we introduce a design of JDBL optimized even-length ABWFBs where the filters are Type-II linear phase filters. The method presented here is a direct time-domain approach that does not involve any parametrization unlike the methods of Tay [14,21]. Our design provides globally optimal solutions. The method is non-iterative where the optimal filter is obtained as an eigenvector of the positive definite matrix. The closed form expressions for the objective function and constraints are given. The performance of the optimal FBs designed by us has been evaluated in image coding and signal denoising applications.

The rest of the paper is organized as follows: The background related with the proposed work is given in Section 2. In Section 3, we formulate the filter design problem as a constrained optimization problem. The derived closed form expressions for objective function and constraints are also given here. Section 4 presents the method to obtain optimal FBs. In Section 5, we present some design examples to demonstrate the effectiveness and flexibility of the proposed method. The section also presents application of the optimal FBs in image compression and denoising. Conclusion and future directions are given in Section 6. The main notations used in this paper are listed in Table 1.

2. Background

In this paper, we consider the design of even-length, two-channel linear phase PR FBs which are known as Type-A FBs. Let $H_0(z)$ and $F_0(z)$ be the analysis lowpass filter (ALF) and synthesis lowpass filter (SLF) of the FB with even-lengths $L_A = 2N$ and $L_S = 2M$, respectively. Throughout the paper, the subscripts A and S stand for analysis and synthesis, respectively. The FB satisfies the PR condition if [24]

$$F_0(z)H_0(z) + F_0(-z)H_0(-z) = 2 \tag{1}$$

Defining the product filter $M(z) = F_0(z)H_0(z)$, the above PR condition (1) can be rewritten as

$$M(z) + M(-z) = 2 \tag{2}$$

In time-domain the PR condition can be written as

Table 1
Important notations.

Symbol	Definition	Symbol	Definition
$H_0(z)$	Analysis lowpass filter (ALF)	J	Objective function
$F_0(z)$	Synthesis lowpass filter (SLF)	γ	Duration-bandwidth
$H_1(z)$	Analysis highpass filter		Trade-off factor
$F_1(z)$	Synthesis highpass filter	K	Regularity index
$M(z)$	Product filter	$\mathbf{D}$	Mean duration matrix
$m(k)$	Impulse response of product filter	$\mathbf{B}$	Mean bandwidth matrix
		$\mathbf{Q}$	Convex combination of matrices **B** and **D**
L_A	Length of ALF		
		$\mathbf{P}$	PR matrix
L_S	Length of SLF	$\mathbf{h}$	Coefficients of symmetric ALF
σ_n^2	Mean-squared duration	$\mathbf{f}$	Coefficients of symmetric SLF
σ_ω^2	Mean-squared bandwidth		
		$\mathbf{V}$	Regularity matrix

Table 2
Duration–bandwidth localization properties.

Filter →	ALF		SLF	
FB ↓	$\sigma_n^2\sigma_\omega^2$	JDBL	$\sigma_n^2\sigma_\omega^2$	JDBL
A-16/28	0.2668	0.4221	0.7700	1.0146
B-16/28	**0.2613**	**0.4130**	**0.7411**	**0.9999**
TFOOL-15/29 [22]	0.2716	0.4319	0.8012	1.0326
MMF-16/28 [24]	0.3430	–	1.4661	–

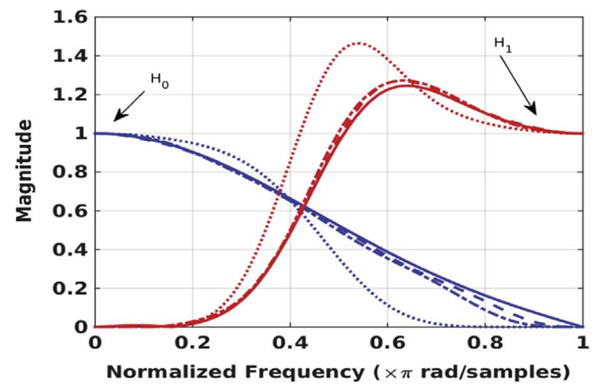

Fig. 1. Frequency response of filter pair [H's] for A–16/28, B–16/28, MMF–16/28 and TFOOL–15/29. Solid line: B–16/28, dashed line: A–16/28, dotted line: MMF FB, dash-dot line: TFOOL–15/29.

$$m(2k) = \sum_n f_0(n)h_0(n-2k) = \delta(k) \tag{3}$$

where $m(k)$ is the impulse response of the product filter $M(z)$ with normalization $m(0) = 1$. The highpass filters $H_1(z)$ and $F_1(z)$ can be obtained from quadrature conjugation of respective lowpass filters.

3. Problem formulation

Consider a Type-II, linear phase, real-valued, finite impulse response (FIR) filter with length $L = 2N$, which is represented by its impulse response, $h(n)$, $0 \le n \le L-1$. The frequency response of the filter can be given by, $H(e^{j\omega}) = e^{(j\omega(L-1)/2)}\widetilde{H}(e^{j\omega})$, where the amplitude response $\widetilde{H}(e^{j\omega})$ is real valued. The amplitude response can be expressed as $\widetilde{H}(e^{j\omega}) = \mathbf{b}^T\mathbf{c}(\omega)$, where, the vectors $\mathbf{b}$ and $\mathbf{c}(\omega)$ have been defined as

$$\mathbf{b} = \sqrt{2}\left[h\left(\frac{L}{2}-1\right), h\left(\frac{L}{2}-2\right), \ldots, h(0)\right]^T \tag{4}$$

$$\mathbf{c}(\omega) = \sqrt{2}\left[\cos\left(\frac{1}{2}\omega\right), \cos\left(\frac{3}{2}\omega\right), \ldots, \cos\left(\frac{L-1}{2}\omega\right)\right]^T \tag{5}$$

The normalizing factor $\sqrt{2}$ is included to have unit energy filter $h(n)$, when the vector $\mathbf{b}$ is constrained to have unit norm.

3.1. Objective function

The mean-squared duration, σ_n^2, and mean-squared bandwidth, σ_ω^2, of a lowpass filter $h(n)$ can be defined by [25],

$$\sigma_n^2 = \frac{1}{E}\sum_{n=-\infty}^{\infty} n^2|h(n)|^2 = \frac{1}{\pi E}\int_0^\pi |\mathbf{H}'(\omega)|^2 d\omega \tag{6}$$

$$\sigma_\omega^2 = \frac{1}{\pi E}\int_0^\pi \omega^2|H(\omega)|^2 d\omega \tag{7}$$

σ_n^2 and σ_ω^2 will subsequently be called duration and bandwidth, respectively. Here, E is the energy of the filter $h(n)$ while $\mathbf{H}'(\omega)$ denotes the derivative of $\mathbf{H}(\omega)$. If the energy of the filter is constrained to be