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Isogeometric collocation: Neumann boundary conditions and contact

L. De Lorenzis[a,*], J.A. Evans[b], T.J.R. Hughes[c], A. Reali[d]

[a] *Institut für Angewandte Mechanik, Technische Universität Braunschweig, Bienroder Weg 87, 38106 Braunschweig, Germany*
[b] *Aerospace Engineering Sciences, University of Colorado Boulder, 1111 Engineering Drive, Boulder, CO 80309-0429, USA*
[c] *Institute for Computational Engineering and Sciences, University of Texas at Austin, 201 East 24th Street, Austin, TX 78712, USA*
[d] *Department of Civil Engineering and Architecture, University of Pavia, via Ferrata 3, 27100, Pavia, Italy*

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Highlights

- Neumann boundary conditions in isogeometric collocation are discussed.
- Two alternative formulations to standard isogeometric collocation are proposed.
- The alternative formulations are more accurate than the standard one for non-uniform meshes.
- A contact formulation for isogeometric collocation is proposed.
- The contact formulation is shown to pass the contact patch test.

Abstract

Isogeometric collocation methods have been proposed recently and their accuracy and efficiency demonstrated for elastostatics and explicit dynamics. This paper addresses two important aspects in the development of the isogeometric collocation technology, namely, the imposition of Neumann boundary conditions and the enforcement of contact constraints, which are both treated within the same framework. It is shown that the strong imposition of Neumann boundary conditions may lead to a significant loss of accuracy in some situations, in particular when non-uniform meshes are used. Two possible remedies are proposed to restore the desired level of accuracy while keeping the computational cost virtually unchanged, i.e. a hybrid collocation–Galerkin approach and an enhanced collocation (EC) approach. A frictionless contact formulation suitable for the collocation framework is further proposed and shown to pass the contact patch test to machine precision. When combined with the EC approach, the formulation is shown to deliver accurate results and to perform robustly also for highly non-uniform meshes. For all the collocation formulations, contact pressures are greater than or equal to zero *pointwise*, in contrast with standard Lagrange finite elements.

Keywords: Isogeometric analysis; Collocation methods; NURBS; Boundary conditions; Contact

1. Introduction

Isogeometric analysis (IGA) was recently introduced by Hughes and coworkers [1,2]. Its main original purpose was to bridge the gap between computer aided design (CAD) and finite element analysis (FEA), thus simplifying

* Corresponding author. Tel.: +49 0 531 391 94350; fax: +49 0 531 391 94399.
E-mail address: l.delorenzis@tu-braunschweig.de (L. De Lorenzis).

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the cost-intensive mesh generation process required for standard FEA and leading to a tighter integration of CAD and FEA tools. In the IGA framework, the same smooth and higher-order basis functions, e.g., non-uniform rational B-splines (NURBS) or T-splines, are used for the representation of the exact CAD geometry and for the approximation of the FEA solution fields.

In addition to the achievement of the original goal, IGA soon turned out to exhibit increased accuracy and robustness on a per-degree-of-freedom basis in comparison to standard finite element methods (FEM) [3,4] and to possess a number of additional attractive features in several areas of computational mechanics. However, smooth higher-order basis functions immediately raised the question of their efficient implementation and, in particular, of computationally efficient quadrature rules. In Galerkin-type formulations, element-wise Gauss quadrature is considered optimal for standard FEM, but sub-optimal for IGA, since it does not exploit the inter-element continuity of its smooth basis functions. Taking advantage of the smoothness across element boundaries, more efficient quadrature rules were developed by Hughes et al. [5] and Auricchio et al. [6]. A natural evolution of this research was the investigation of isogeometric collocation (IGA-C) methods [7,8], which can be interpreted as a one point quadrature rule in the IGA context.

As opposed to Galerkin formulations, collocation is based on the discretization of the strong form of the governing partial differential equations. This is only possible if basis functions of sufficient smoothness and, therefore, high order are adopted. This requirement is naturally fulfilled by the shape functions used in IGA, which feature tailorable order and inter-element continuity. Furthermore, the IGA framework allows domains of arbitrary geometric and topological complexity to be discretized, possibly in tight integration with CAD. In IGA-C, only one point evaluation per control point, or "node", is required at a collocation point. This implies the minimization of the computational effort with respect to quadrature and is a major advantage over Galerkin-type methods, especially for applications where efficiency is directly related to the cost of quadrature. A few significant examples are explicit structural dynamics, where the computational cost is dominated by stress divergence evaluations at quadrature points for the calculation of the residual force vector, multiscale methods based on nested solution schemes such as FE^2, where a microscale boundary value problem is solved for each quadrature point of the macroscale, and stochastic approximation procedures such as the stochastic FEM, where the residual vector must be evaluated a large number of times.

One point quadrature in conjunction with low-order quadrilateral and hexahedral finite elements is already extensively (if not exclusively) used in crash dynamics and metal forming. This minimizes memory requirements and the number of constitutive evaluations and enables the efficient computation of very large problems. However, one point quadrature with standard basis functions gives rise to zero-energy hourglass modes and rank deficient system matrices [9]. The problem is addressed by introducing artificial viscous and/or elastic stabilization mechanisms, whose parameters often require tuning by time consuming and computationally expensive sensitivity studies. Conversely, it can be shown that for quadratic and higher-order NURBS, with uniform knot vectors and a suitable choice of the collocation points, the discrete Laplace operator produced by collocation is rank sufficient in all dimensions. It follows that the elasticity operator is also rank sufficient and, in particular, there are no hourglass modes. Thus IGA-C can be viewed as a one point quadrature scheme that is rank sufficient [8,10]. Hence, IGA collocation methods eliminate the need for ad-hoc stabilization techniques. Furthermore, they show great promise for the development of higher-order accurate time integration schemes [8] as well as for the development of locking-free beam, plate and shell elements [11,12].

Before a brief literature review, it is worth mentioning that collocation techniques are also quite often adopted within the framework of meshless methods, see, e.g., [13–20]. However, the IGA-C method is *not* a meshless method; here the isoparametric concept is adopted. Meshes are employed that either have a tensor product structure or are locally refined (depending on whether NURBS or T-splines or hierarchical B-splines are adopted). In this paper, only one-patch domains with NURBS basis functions are used in the examples. However, the extension to multi-patch NURBS domains is conceptually straightforward to obtain, at least as long as patches with matching discretizations are adopted, whereas a collocation-based framework for hierarchical B-spline discretizations has been presented by Schillinger et al. [10].

Only a few investigations of IGA-C have been conducted thus far. Auricchio et al. [7] developed the first one-dimensional theoretical analysis of the method, which served the dual purpose of providing the theoretical background and guiding the selection of collocation points. They presented numerical tests on simple elliptic problems in one, two and three dimensions, demonstrated the accuracy of the method, studied the behavior of the discrete eigenspectrum