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Improved numerical integration for locking treatment in isogeometric structural elements. Part II: Plates and shells

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Highlights

- We model Reissner–Mindlin isogeometric plates and shells.
- We examine membrane and shear locking in bending dominated problems.
- Higher continuity elements exhibit superior accuracy when no locking occurs.
- We extend one-dimensional reduced quadrature rules to two-dimensional rules.
- We assess the performance of the schemes using the shell obstacle course problems.

Abstract

B-spline reduced quadrature rules are proposed in the context of isogeometric analysis. When performing a full Gaussian integration, the high regularity provided by spline basis functions strengthens the locking phenomena and deteriorates the performance of Reissner–Mindlin elements. The uni-dimensional B-spline-based quadrature rules, given in a previous paper (part I), are extended to multi-dimensional problems such as plates and shells. The improved reduced integration schemes are constructed using a tensor product of the uni-dimensional schemes. A single numerical quadrature is performed for bending, transverse shear and membrane terms, without introducing Hourglass modes. The proposed isogeometric reduced elements are free from membrane and transverse shear locking. Convergence is first assessed in plate problems with several aspect ratios and then in the shell obstacle course problems. The resulting under-integrated elements exhibit better accuracy and computational efficiency.

Keywords: Isogeometric analysis; B-splines/NURBS; Numerical locking; Selective/reduced integration; Reissner–Mindlin shells

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1. Introduction

B-spline/NURBS-based isogeometric analysis has shown potentially attractive properties for large deformation problems such as automobile crashworthiness. Lipton et al. [1] have demonstrated that robustness of isogeometric elements increases with order. High order Lagrange elements are sensitive to mesh distortion that generally prevents their use in large deformation problems. B-spline/NURBS elements appear to be increasingly robust when higher order and higher continuity functions are used in the context of structural analysis. Moreover, structural analysis is a branch of computational engineering that generally requires a surface description of the CAD geometry. Since CAD software makes use of boundary representations to model geometrical objects, bridging design with analysis results in a simpler process. The isogeometric analysis (IGA) process becomes thus an attractive alternative to finite element analysis (FEA) for structure computations.

This recent concept, first proposed by T.J.R. Hughes et al. [2], uses the exact geometry representation, as it is created in CAD software. The faceted geometry generated in FEA, which exhibits poor properties when, e.g. assembling several parts in a model or simulating contact problems, is replaced by the exact and smooth geometry in IGA. Moreover, k-refinement [2,3] preserves this smoothness when refining or elevating the order of the geometry. Considering a basis of order p, k-refinement achieves a C^{p-1} regularity while p-refinement only keeps the initial continuity across the elements.

The two main theories in shell analysis – based on the Reissner–Mindlin and Kirchhoff–Love hypothesis – have been intensively studied for decades. While the second theory is only valid for thin shells, the shear deformable theory can be used to describe both thick and thin shells. Kirchhoff–Love shells require at least C^1 regularity which is not achieved for standard Lagrange finite elements. The development of C^1 shell elements has proved difficult so that the C^0 shear deformable theory proves more popular for structural FEA. B-spline/NURBS geometries [4] provide the required continuity throughout a whole patch, which makes the development of isogeometric Kirchhoff–Love elements simpler [5]. However, the classical C^0 connection of multiple patches is a major difficulty that requires additional techniques to artificially impose a higher continuity [6]. Structural analysis in the context of spline-based IGA is an area of active research. Benson et al. [7] and more recently Dornisch et al. [8] have proposed Reissner–Mindlin shell elements. Kiendl et al. [5,6] and Benson et al. [7] have worked on Kirchhoff–Love shells. Initially, B-spline or NURBS basis functions are employed but more sophisticated polynomial splines over hierarchical T-meshes have been combined to shell theories [9]. Moreover, recent developments on blended isogeometric shells that benefit from the assets of both hypothesis are presented in [10].

Since IGA provides a smooth geometry and can guarantee at least C^1 regularity across the elements, the exact normal vector of the shell mid-surface can be computed at each point of the geometry, excepted for reduced continuity points. However, a major drawback is that the control points are not, in general, located on the shell. Therefore, when using Reissner–Mindlin hypothesis, adapted methods must be employed to define a unique director vector, at each control point, for the interpolation of the rotations. Several methods are presented and compared in [8,11]. This is an important aspect since the interpolation of the directors can deteriorate the quality of the results. The nodal values have to be well chosen and open questions still remain.

Low order Lagrange structural elements, based on Timoshenko/Mindlin hypothesis, suffer from severe membrane and shear locking [12–15]. These pathologies arise from field inconsistency and exhibit the incapacity of the basis functions to represent shearless and inextensional bending behaviours. Echter et al. [16] have shown that isogeometric finite elements are not free from numerical locking. However, Rank et al. [17] observed that shear locking is significantly reduced in thin Reissner–Mindlin plates with a Lagrange polynomial order higher than six. To overcome this issue, an intuitive idea consists in increasing the polynomial order of the shape functions. However, such a strategy becomes time consuming while using very high orders. Moreover, most of the classical geometries can be modelled with order two. Therefore, this work has been restricted to bivariate quadratic and cubic spline functions.

Numerical locking in structural analysis has been an active domain of research for many years. Different methods have emerged from this work. These pathologies can be alleviated with a field-consistent approach that uses distinct orders of interpolation [18–20] in the membrane and shear terms, or by redefining the link between displacements and membrane/shear strains. Assumed strains and $\bar{\text{B}}$ projection methods [21–25] exhibit a good accuracy and robustness. Mixed and hybrid formulations [20,26,27] also remove the numerical locking.

Selective and reduced integration techniques [28–36] are another class of methods that successfully reduce the numerical locking. The excessive bending stiffness of the fully integrated shell element is reduced lowering – by