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Image-based goal-oriented adaptive isogeometric analysis with application to the micro-mechanical modeling of trabecular bone

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Highlights

- A smooth geometry is reconstructed from voxel data using a B-spline approximation.
- A goal-adaptive isogeometric finite cell method is used to compute elastic properties.
- A tessellation-based procedure is proposed to accurately integrate trimmed elements.
- The proposed methodology is applied to μCT-scan data of a trabecular bone specimen.

Abstract

Isogeometric analysis (IGA) of geometrically complex three-dimensional objects is possible when used in combination with the Finite Cell method (FCM). In this contribution we propose a computational methodology to automatically analyze the effective elastic properties of scan-based volumetric objects of arbitrary geometric and topological complexity. The first step is the reconstruction of a smooth geometry from scan-based voxel data using a B-spline level set function. The second step is a goal-oriented adaptive isogeometric linear elastic analysis. Elements are selected for refinement using dual-weighted residual shape function indicators, and hierarchical splines are employed to construct locally refined spline spaces. The proposed methodology is studied in detail for various numerical test cases, including the computation of the effective Young's modulus of a trabecular bone micro-structure reproduced from μCT-scan data.

Keywords: Isogeometric analysis; Finite cell method; Hierarchical splines; Goal-oriented error estimation and adaptivity; Image-based analysis; Trabecular bone micro-structures

1. Introduction

Isogeometric analysis (IGA) was introduced by Hughes et al. in 2005 [1] as a novel analysis paradigm aiming at the unification of the fields of computer aided (geometric) design (CAD) and finite element analysis (FEA), see also [2].

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The fundamental idea of IGA is to directly use the CAD parametrization of a geometric design for the purpose of analysis. Consequently, in contrast to the finite element method, no geometry clean-up or meshing operations are required. This rigorous elimination of the meshing step benefits the design-through-analysis process, especially for complex designs. Additionally, the spline basis functions inherited from CAD have various advantageous properties compared to the basis functions used in finite elements. The smoothness of higher-order spline basis functions is the most prominent of such advantageous properties. Due to its merits, isogeometric analysis has been applied in the context of both Galerkin-based and collocation-based (*e.g.* [3,4]) analysis for a wide variety of applications, encompassing the domains of fluid dynamics, (non-linear) solid mechanics, and multi-physics modeling. Finite element data structures have been developed to facilitate the implementation of the isogeometric analysis paradigm in existing finite element codes, see *e.g.* [5–7].

Recently, isogeometric analysis has successfully been applied for the discretization of a variety of problems on geometrically and topologically complex volumetric domains by using it in conjunction with the finite cell method [8,9]. In this method a trivariate tensor-product domain is created in which the complex domain of interest is immersed [10,11]. The B-spline (or NURBS) basis functions required for the construction of trial and test spaces to be used in combination with a Galerkin problem are initially constructed over the structured domain, after which they are restricted to the domain of interest. Since an underlying structured mesh is available, local refinements can be obtained using hierarchical splines, see *e.g.* [12].

A particularly interesting application area of the finite cell method is the analysis of image-based (or scan-based) geometric models [13]. Since the finite cell method does not require a conforming mesh, cumbersome meshing operations can be circumvented. Instead, the image data is used on a sub-element level to trim the structured domain to the immersed domain of interest. The image-based finite cell method has been used for *e.g.* the analysis of metal foams [14], the validation of in vitro bone experiments [15], and trabecular bone micro-structures [11] (see [13] for an overview).

In this contribution we study the application of the isogeometric finite cell method to the elastic analysis of trabecular bone specimens. Numerical analysis of trabecular bone plays an important role in efficacy studies of osteoporosis treatments. In the last few decades micro-scale finite element techniques have been proposed that use a voxel conversion technique to represent the bone micro-structure with brick elements (see *e.g.* [16–18]). Although such analyses can well predict bone stiffness in some clinical research studies, the disadvantages associated with such analyses prevent wider application. Most importantly, due to the non-smoothness of the geometric model, microscopic stresses cannot be represented accurately. Alternative finite element techniques, *e.g.* based on tetrahedral meshes (see *e.g.* [19–21]), have been developed to ameliorate the deficiencies of analyses using brick elements. However, due to the poor performance of linear tetrahedron elements, and the computational expenses of second order tetrahedrons, these are not commonly used. Based on its proven advantages, it is anticipated that isogeometric analysis can render accurate and reliable computational results for the elastic analysis of trabecular bone micro-structures. In this contribution we outline a computational methodology to automatically perform the elastic analysis of scan-based geometric models with a high degree of computational accuracy.

One of the novel contributions of this manuscript is the introduction and analysis of a B-spline-based approximation strategy for gray scale voxel data, which provides an implicit definition of the scan-based geometry with a smooth internal boundary. The second novelty of this work is the usage of goal-oriented adaptive analysis [22] in the context of the scan-based isogeometric finite cell method. This aspect of our work builds on the recent work in [23], where goal-oriented isogeometric analysis is considered in the context of tensor-product B-splines. The usage of adaptive methods is of paramount importance in the context of three-dimensional scan-based geometric models, since manual refinement operations are impractical. These operations involve both the aspect of identifying regions with high contributions to the error in the quantity of interest, and the actual refinement of the elements in these regions. In addition to these two main contributions, we outline various details of the finite cell method that are specific to our work. Most importantly, we employ and study a simplex-based tessellation of the elements that intersect the boundary of the physical domain.

The computational methodology proposed in this work is first studied for two-dimensional test cases, for which high accuracy reference solutions can be computed. This allows us to perform detailed convergence studies, and to assess the quality of our error estimates. Subsequently we apply the methodology to the elastic analysis of a realistic data set obtained by a μCT-scan of a trabecular bone micro-structure. Our terminology is tailored to the three-dimensional setting. The words “pixel” and “voxel” are for example used synonymously in the two-dimensional