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An extended isogeometric thin shell analysis based on Kirchhoff–Love theory

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Highlights

- An extended isogeometric element formulation (XIGA) for analysis of through-the-thickness cracks in thin shell structures.
- XIGA can reproduce the singular field near the crack tip and the discontinuities across the crack.
- NURBS basis possesses C^1-continuity required by Kirchhoff–Love theory without additional rotational degrees of freedom.

Abstract

An extended isogeometric element formulation (XIGA) for analysis of through-the-thickness cracks in thin shell structures is developed. The discretization is based on Non-Uniform Rational B-Splines (NURBS). The proposed XIGA formulation can reproduce the singular field near the crack tip and the discontinuities across the crack. It is based on the Kirchhoff–Love theory where C^1-continuity of the displacement field is required. This condition is satisfied by the NURBS basis functions. Hence, the formulation eliminates the need of rotational degrees of freedom or the discretization of the director field facilitating the enrichment strategy. The performance and validity of the formulation is tested by several benchmark examples.

Keywords: Thin shells; Fracture mechanics; Isogeometric analysis; NURBS; XFEM; XIGA

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1. Introduction

IsoGeometric Analysis (IGA) was introduced by [1] in order to integrate Computer Aided Design (CAD) and Computer Aided Engineering (CAE). The idea is to use CAD basis functions also in numerical analysis. While the finite element method (FEM) with Lagrangian basis function is most popular in CAE, the most common CAD basis functions are Non-Uniform Rational B-Splines (NURBS). Other recent approaches in IGA are based on other basis functions such as T-splines [2–4], polynomial splines over hierarchical T-meshes (PHT-splines) [5–9], hierarchical B-splines/NURBS [10,11] and piecewise polynomial finite element basis functions over hierarchical T-meshes [12]. The isogeometric approach was first developed for solids [13,14] and then extended to structural elements [15–19], fluids [20–23], as well as to contact problems and cohesive fracture problems [24–27].

The analysis of thin shell structures is of vital importance in many engineering applications such as aircraft fuselages, storage tanks, ship hulls, and pipes. A variety of shell elements has been developed to analyze shell structures which can be classified by the thickness of the shell and the curvature of the mid-surface [28]. Depending on the thickness, shell elements can be categorized into thin shell elements [29–33] and thick shell elements [34–36]. Thin shell elements are based on the Kirchhoff–Love (KL) theory [37] where transverse shear deformations are negligible. The KL-theory requires C^1-continuity of the displacement field, which is difficult to achieve for free-form geometries when a Lagrangian-type basis is used. Thin shell theory requires the approximation of the deformation to have second-order square integrable derivatives. Using higher continuous formulations in the context of thin shell analysis based on KL-theory avoids the use of rotational degrees of freedom or discretization of the director field. A formulation that only discretizes the mid-surface position and naturally fulfills the KL-theory constraint by using a higher continuous formulation was first proposed in the context of mesh-free methods by [38–40]. These formulations include modeling of fracture through partition-of-unity enrichment [41–43]. In the context of IGA, [44,45] proposed thin shell formulations based on KL-theory taking advantage of the higher-order continuity of the NURBS basis functions. The formulation in [44] was extended by [8] to efficiently account for h-adaptive refinement.

An approach combining the extended finite element method (XFEM) and IGA for fracture in continua was proposed by [46–48]. Instead of modeling the crack in IGA through partition-of-unity enrichment, Verhoosel et al. [49] applied the concept of knot insertion to create a discontinuous displacement field. An XFEM-shell formulation (for thick shells) based on Lagrange polynomials was proposed by [50]. However, the formulation requires many degrees of freedom (DOFs) per node. A simpler approach based on overlapping paired elements and discrete KL-theory was suggested by [51] and applied to numerous interesting examples in [52]. In [53], a phantom node method [54] for fracturing thin structures was proposed. The computation of the interaction integral was derived for both Kirchhoff–Love as well as Mindlin–Reissner theory. In contrast to the approach in [51], this formulation allows the crack tip to be located inside an element [55] allowing for much coarser discretizations.

In this manuscript, we present an extended isogeometric analysis (XIGA) method for cracked thin shell structures. The method is capable of handling efficiently through-the-thickness cracks in thin shells. NURBS basis functions ensure the C^1-continuity of the generalized displacements. Since no rotational degrees of freedom are used, the complexity of the enrichment strategy and the computational cost are significantly reduced. The good performance of the method is confirmed by numerical examples.

The content of the paper is outlined as follows. In Section 2, we describe the thin shell formulation. The XIGA thin shell formulation is derived in Section 3. Section 4 presents several benchmark examples before the paper closes with concluding remarks.

2. Thin shell formulation

In thin shell theory, the three-dimensional continuum description is reduced to the shell mid-surface, and the transverse normal stress is neglected. Furthermore, the Kirchhoff–Love theory assumes that the shell cross-section remains normal to its mid-surface in the deformed configuration, which implies that the strain is assumed to be linear through the thickness and the transverse shear strains are zero. In this paper, we have adopted the thin-shell formulation for intact continua from [44,8] and extended it to fracture problems.