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Isogeometric mortar methods

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Highlights

- Isogeometric mortar methods from the theoretical and numerical point of view.
- Three well motivated choices of dual spaces with a different degree.
- Two suitable Lagrange multiplier spaces.
- One choice, reasonable at the first glance, but unstable.
- A numerical example coupling non-matching bodies.

Abstract

The application of mortar methods in the framework of isogeometric analysis is investigated theoretically as well as numerically. For the Lagrange multiplier two choices of uniformly stable spaces are presented, both of them are spline spaces but of a different degree. In one case, we consider an equal order pairing for which a cross point modification based on a local degree reduction is required. In the other case, the degree of the dual space is reduced by two compared to the primal. This pairing is proven to be inf–sup stable without any necessary cross point modification. Several numerical examples confirm the theoretical results and illustrate additional aspects.
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1. Introduction

The name isogeometric analysis was introduced in 2005 by Hughes et al. in [1]. Nowadays it includes a family of methods, normally called isogeometric methods, that use B-Splines and non-uniform rational B-Splines (NURBS) as basis functions to construct numerical approximations of partial differential equations (PDEs). Originally,

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isogeometric analysis follows the isoparametric paradigm, i.e., the geometry is represented by functions which are used to approximate the PDE. In [2], it was shown that this concept can be relaxed, also allowing NURBS for the parametrization and B-Splines defined on the same mesh for the approximation of the PDE.

With isogeometric methods, the computational domain is generally split into patches. Within this framework, techniques to couple the numerical solution on different patches are required. To retain the flexibility of the meshes at the interfaces, weak coupling methods are favorable in contrast to strong point-wise couplings. Thus it is interesting to consider mortar methods, which offer a flexible approach to domain decomposition, originally applied in spectral and finite element methods. Mortar methods have been successfully investigated in the finite element context for over two decades, [3–7], for a mathematical overview, see [8]. Further applications of the mortar methods include contact problems, [9–13], and interface problems, e.g., in multi-physics applications, [14].

The isogeometric analysis, [15,16], is currently a very active research area. It is attractive for a large variety of applications and there already exists a fair amount of mathematically sound results, recently collected in [2]. Besides variational approaches, the global smoothness of splines also allows the use of collocation methods, see [17].

In several articles, the coupling of multipatch geometries has been investigated, [18–22], and successful applications of the mortar method are shown in [23–25]. Additionally the use of mortar methods in contact simulations, where isogeometric methods have some advantages over finite element methods, was considered in [26–31].

The important point of an isogeometric mortar method is the choice of the Lagrange multiplier. From the classical mortar theory, two abstract requirements for the Lagrange multiplier space are given. One is the sufficient approximation order, the other is the requirement of an inf–sup stability. For a primal space of splines of degree p, we investigate three different degrees for the Lagrange multiplier: p, $p-1$ and $p-2$. Each choice is from some point of view natural but has quite different characteristic features.

This article is structured as follows. In Section 2, we recall basic properties of isogeometric methods. The isogeometric mortar methods is then defined in Section 3. In Section 4, we complete the definition of our mortar methods by explicitly detailing three different types of Lagrange multipliers. The theoretical results are investigated numerically in Section 5, where also additional aspects are considered.

2. B-Splines and NURBS basics

In this section, we give a brief overview on the isogeometric functions and introduce some notations and concepts which are used throughout the paper. For more details, we refer to the classical literature [15,32–34]. Firstly, we introduce B-Splines in the one-dimensional case and recall some of their basic properties. Secondly, we extend these definitions to the multi-dimensional case and introduce NURBS and then NURBS parametrizations.

2.1. Univariate B-Splines

Let us denote by p the degree of the univariate B-Splines and by Ξ an open univariate knot vector, where the first and last entries are repeated $(p+1)$-times, i.e.,

$$\Xi = \{0 = \xi_1 = \cdots = \xi_{p+1} < \xi_{p+2} \le \cdots \le \xi_n < \xi_{n+1} = \cdots = \xi_{n+p+1} = 1\}.$$

Let us define $Z = \{\zeta_1,\ \zeta_2, \ldots, \zeta_E\}$ as the knot vector without any repetition, also called breakpoint vector. For each breakpoint ζ_j of Z, we define its multiplicity m_j as its number of repetitions in Ξ. The elements in Z form a partition of the parametric interval $(0, 1)$, i.e., a mesh.

We denote by $\widehat{B}_i^p(\zeta),\ i = 1, \ldots, n$, the collection of B-Splines defined on Ξ and by $S^p(\Xi) = \mathrm{span}\{\widehat{B}_i^p(\zeta),\ i = 1, \ldots, n\}$ the corresponding spline space.

We recall hereafter some important properties of the univariate B-Splines. Each $\widehat{B}_i^p$ is a piecewise positive polynomial of degree p and has a local support, i.e., $\widehat{B}_i^p$ is non-zero only on at most $p+1$ elements and $\mathrm{supp}\,\widehat{B}_i^p = [\xi_i, \xi_{i+p+1}]$. Consequently on $[\zeta_i,\ \zeta_{i+1}]$ at most $p+1$ basis functions have non-zero values. The inter-element continuity is defined by the breakpoint multiplicity. More precisely, we have that the basis functions are C^{p-m_j} at each $\zeta_j \in Z$.

Assuming that $S^p(\Xi) \subset C^0(0, 1)$ (i.e., $m_j \le p,\ j = 2, \ldots, E-1$), and let $\Xi' = \{\xi_2, \ldots, \xi_{n+p}\}$, then the derivation operator $\partial_\zeta : S^p(\Xi) \to S^{p-1}(\Xi')$ is linear and surjective, see [2,34].

For spline spaces, different refinement strategies are available. Further knots can be inserted (h-refinement), the degree can be elevated (p-refinement) and a combination of both is possible (k-refinement). We refer to [15,32] for

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