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Isogeometric numerical dispersion analysis for two-dimensional elastic wave propagation

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Highlights

- A numerical dispersion analysis for the linear elastodynamics equations is performed.
- The numerical approximation is carried out with NURBS-based Isogeometric Analysis.
- Anisotropic curves and errors of compressional and shear wave velocities are provided.
- The dispersion analysis is compared for B-splines and NURBS of different regularity.

Abstract

In this paper, we carry out a numerical dispersion analysis for the linear two-dimensional elastodynamics equations approximated by means of NURBS-based Isogeometric Analysis in the framework of the Galerkin method; specifically, we consider the analysis of harmonic plane waves in an isotropic and homogeneous elastic medium. We compare and discuss the errors associated with the compressional and shear wave velocities and we provide the anisotropic curves for numerical approximations obtained by considering B-spline and NURBS basis functions of different regularity, namely globally C^0- and C^{p-1}-continuous, p being the polynomial degree. We conclude our analysis by numerically simulating the seismic wave propagation in a sinusoidal shaped valley with discontinuous elastic parameters across an internal interface.

Keywords: Elastodynamics; Elastic waves propagation; Isogeometric analysis; Numerical dispersion analysis

1. Introduction

In the last decade, Isogeometric Analysis (IGA) [1,2] has emerged as a methodology aiming at encapsulating the exact geometrical representation of the computational domain, namely the field of Computational Geometry (see

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e.g. [3]), into the numerical approximation of Partial Differential Equations (PDEs). This integration is made possible by the use of the same basis functions considered for the geometrical representation also for the approximation of the unknown solution fields of the PDEs, introducing the so-called Isogeometric concept [2]. B-spline and Non Uniform Rational B-spline (NURBS) basis functions [4] have mostly been considered for the IGA methodology being the foundations of Computer Aided Design (CAD) systems, even if other geometrical representations as T-splines [5] have been employed as well for their flexibility; see e.g. [6]. So far, NURBS-based IGA has been mostly used in the framework of the Galerkin method [1,2,7], even if collocation techniques are recently receiving growing attention [8, 9]. The advantages of the IGA methodology in terms of the "exact" geometrical representation have been exploited in several applications, as e.g. structural mechanics [2,10,11] and fluid dynamics [12–14] among the most common. Moreover, the use of B-spline and NURBS basis functions in IGA possess several advantages in the numerical approximation of PDEs regardless of the geometrical considerations, as highlighted, e.g., in fluid dynamics [15], structural dynamics [16,17], high-order PDEs [18], and phase field problems [19,20]. Such advantages include the possibility of using globally C^{p-1}-continuous basis functions, p being the polynomial degree, and the k-refinement strategy, a procedure for the "enrichment" of the discrete function spaces peculiar of B-splines and NURBS for which the degree and global continuity of the basis functions are increased; see e.g. [21,22]. In particular, the use of globally C^{p-1}-continuous NURBS basis functions has been shown to be superior to its Finite Elements counterpart of polynomial degree p by means of extensive spectrum and dissipation analyses, both in terms of analytical and numerical results for $1D$, $2D$, and $3D$ structural, vibration, acoustic, and wave propagation problems [1,16,17,21–25].

Numerical (grid) dispersion analysis for the linear elastodynamics equations, i.e for linear wave propagation in an elastic medium, is often used to assess the accuracy of numerical schemes for applications in civil, geophysics, and earthquake engineering. Such analysis has been extensively carried out for the Finite Elements method [26,27], Discontinuous Galerkin methods [28–32], and the Spectral (element) method [33–35], including non-conforming high-order discretizations [36]. In [23] a numerical dispersion analysis has been performed for NURBS-based IGA for the Helmholtz equation in the $1D$ setting on an infinite line, including linear and p- and k-refined quadratic approximations. This analysis has been extended in [25] to higher degree NURBS basis functions for vibration problems of rods and beams of finite length. In addition, a numerical dispersion analysis for $2D$ vibration problems described by the Helmholtz equation is reported in [25] for the special case of a bilinear approximation; the associated anisotropic (dispersion) curve is also reported for this case only.

In this respect, in this paper we propose a numerical dispersion analysis for the elastodynamics equations in $2D$, specifically for the linear wave propagation in an isotropic elastic medium, in terms of the spatial approximation by means of NURBS-based IGA in the framework of the Galerkin method. We report for the first time the anisotropic curves and errors associated with the compressional and shear wave velocities in the elastic medium by considering both B-spline and NURBS basis functions and different material properties (characterized by their Poisson ratio). Specifically, in our numerical comparison, we consider B-spline and NURBS basis functions defined over uniform knot vectors with different polynomial degrees p with particular emphasis on their regularity properties, i.e. their global C^0- or C^{p-1}-continuity in the computational domain (this corresponds to either p- or k-refinement, see [21,22]); we also study the case of a section of an annulus geometrically represented by NURBS. Our dispersion analysis is based on the procedure proposed in [35] for the coherent comparison of numerical schemes in bounded computational domains for different wave directions without the need to strongly enforce periodic boundary conditions. Specifically, we adapt the approach of [35], originally developed for Spectral (element) methods, to NURBS-based IGA in the framework of the Galerkin method with the aim of consistently comparing the results obtained with basis functions of different polynomial degrees p and global C^0- and C^{p-1}-continuity.

We conclude our analysis by numerically simulating a seismic event, i.e. an elastic wave propagation problem, in a $2D$ portion of the earth mantle embedding a sinusoidal type valley. The latter is delimited by an internal interface, which separates two regions with discontinuous material parameters (different media); such configuration is suitably represented by means of C^0/C^1-continuous B-spline basis functions. For the numerical simulation of this seismic event, we use NURBS-based IGA for the spatial approximation and the generalized-α method [37] for the time discretization with a fully implicit scheme. Through this example we numerically highlight the suitability of NURBS-based IGA to solve elastodynamics problems with discontinuous material properties across internal interfaces.

The paper is organized as follows. In Section 2 we briefly recall the linear elastodynamics model used in seismic applications. Section 3 introduces to B-spline and NURBS basis functions, geometrical representations, and the Isogeometric concept. In Section 4 we discuss the spatial approximation of the elastodynamics equations by means