ARTICLE IN PRESS
JID: ADES [m5G;August 4, 2016;9:43]
Advances in Engineering Software 000 (2016) 1–8
Contents lists available at ScienceDirect
Advances in Engineering Software
journal homepage: www.elsevier.com/locate/advengsoft

Pre-stressed rubber material constant estimation for resilient wheel application

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ARTICLE INFO

Article history:
Received 29 January 2016
Revised 3 June 2016
Accepted 18 July 2016
Available online xxx

Keywords:
Identification
Rubber
Viscous-elastic constant
Pre-press
Dynamic loading

ABSTRACT

This paper describes the methodology and results of the complex modulus estimation of rubber segments pressed between the disk and the rim of a rubber-damped railway wheel. The solution comes out from the method proposed and validated at rubber constant identification on the case of the steel beam with rubber layer. In the vicinity of eigenfrequency its dependence to Young modulus is quasilinear and can be solved effectively by the gradient method. The similar behavior showed itself also for dependence of the modal damping constant on the damping coefficient. The ascertained results of tuned rubber material constants, i.e. the modulus of elasticity and a loss factor, of the rail rubber segments are very valuable for prediction of modal behavior of the new resilient wheels and they are qualitatively in accordance with the behavior of hard synthetic rubbers.

1. Introduction

The main goal of the study was to develop a method for estimation of the frequency dependence of material constants of the hard synthetic rubbers. It was motivated the need to identify material constants, i.e. Young modulus and loss factor, of rubber segments pressed between the disk and the rim of a resilient wheel [1,2]. The rubber segments are pre-pressed during the production of the wheel. Hard synthetic rubber materials exhibit complex thermal-frequency behavior with nonlinear dependence on static preload. The better estimation of the rubber material constants helps to predict and optimize the vibro-acoustic behavior of the wheel.

In general, the updating techniques using the optimization of the structural parameters based on the fitting of numerical to experimental characteristics of the structures are very favorite tool for an estimation of different material parameters in the literature [e.g. 3–10]. Proposed methodology of parametric identification of rubber constants comes from the standard experimental procedure designated by the Oberst beam [11,12], that evaluate the frequency dependence of the material based on the vibrations of a cantilever beam that consists of a metal and a rubber layer.

The novelty of the proposed methodology is in an extension of the standard method to more general and damped structures such as the railway wheel with rubber segments where the numerical solution by the finite element method using space discretization is needed. Furthermore we propose the methodology for parametric tuning of the loss factor of the rubber. The special proportional model was designed as the damping rubber model that as proved experimentally fits as a proper material damping model in a bandwise divided frequency range of interest and also suits to parametrization of the damping model for the numerical tuning of the real part of the wheel calculated eigenvalues to their experimental counterparts. Thanks to the extension the frequency dependence of the Young modulus and loss factors of largely pre-strained rubber segments of the resilient railway wheel was ascertained.

At first a dependence of the material constants of rubber layer on the eigenvalues of the composed beam and accuracy of their identification by the updating technique was numerical studied. This case study brought the experience how to apply this approach on a case of the railway wheel with imposed rubber in a pre-stressed condition [13].

Then the proposed estimation method of the complex modulus of elasticity of rubber for resilient railway wheel is described. It is based on the tuning of rubber constants of a finite element (FE) wheel model according to the results of natural frequencies and mode shapes of the wheel ascertained from the experiment, e.g. [14–17]. The experimental modal analysis of the wheel took place at room temperature in a dynamic laboratory. The identification of eigenvalues and mode shapes of the wheel was made separately for excitations both in radial and in axial direction. Finally, the results of the rubber material constants are presented and discussed.

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http://dx.doi.org/10.1016/j.advengsoft.2016.07.009
0965-9978/

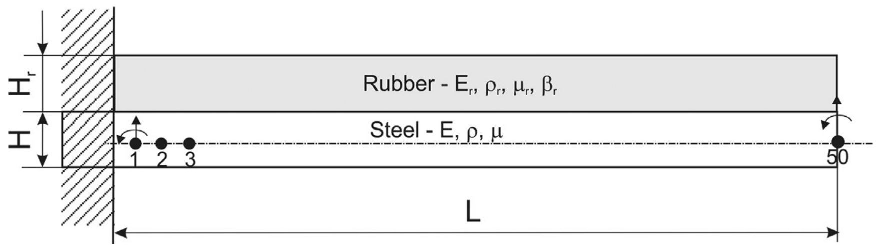

Fig. 1. Scheme of the finite element model of the composed beam.

2. Case study of parametric estimation of rubber material constants

Eigenvalue problem equation for free vibration of a damped non-conservative model can be converted to the quadratic eigenvalue problem

$$(\mathbf{K} + \lambda_i \mathbf{B} + \lambda_i^2 \mathbf{M})\mathbf{x}_i = \mathbf{0}, \tag{1}$$

where **K, B** and **M** are the stiffness, damping and mass matrices, λ_i represents a set of eigenvalues and $\mathbf{x}_i$ their eigenvectors. Then *i*th eigenvalue can be written as $\lambda_i = -b_i \pm j\omega_i$, where b_i is a modal damping constant, ω_i is a natural frequency and the auxiliary quantity of damping ratio is defined as $\zeta_i = b_i/\omega_i$.

We assume here that the difference between the natural frequency of the damped and undamped system is negligible for calculating the damping ratio.

Damping matrix of rubber $\mathbf{B}_r$ is described by a special model of proportional damping [18] as

$$\mathbf{B}_r = \beta_r \mathbf{K}_r \tag{2}$$

where β_r is the stiffness coefficient of damping, $\mathbf{K}_r$ is the stiffness matrix of rubber segments.

The relationship between the loss factor and the stiffness coefficient of damping is

$$\tan(\delta) = \beta_r \omega \tag{3}$$

where ω is the vibration frequency.

At linear viscous-elastic model of rubber, an elasticity modulus is characterized by a complex number $E_r^* = E_r + jE_{rD}$ where E_r is the Young modulus and E_{rD} loss modulus. Then loss factor of rubber is defined as

$$\tan(\delta) = \frac{E_{rD}}{E_r}. \tag{4}$$

Damping matrix of rubber $\mathbf{B}_r$ only contributes to the overall damping matrix of the wheel **B** and therefore, global matrix **B** is not proportional to the global stiffness matrix **K**. Therefore, solving the quadratic eigenvalue problem (2) leads to a complex eigenvalues.

For evaluation of accuracy and convergence of the proposed optimization, we built the numerical model of a cantilever steel beam with rubber layer in the MATLAB environment.

The finite element beam model (Fig. 1) consisted of 50 elements and was based on the Euler theory. Material and geometric parameters of the steel beam and the rubber layer were: $E = 2.1.10^{11}$ Pa, $\rho = 7850\ \mathrm{kg\,m^{-3}}$, $\mu = 0.3$, L = 0.7 m, width B = 0.050 m, H = 0.05 m and $\rho_r = 1300\ \mathrm{kg\,m^{-3}}$, $\mu_r = 0.49$, $L_r = 0.7$ m, $B_r = 0.050$ m, $H_r = 0.05$ m, respectively.

Method of identification of Young's modulus E_r and the damping coefficient β_r with respect to the influence of rubber layer on the dynamics of the steel beam was proposed to ascertain these constants by minimization of a cost functions defined both as difference between imaginary ω_1 and as difference between real part b_1 of the first eigenvalue of the reference composed beam and their approximated counterparts computed for the model with optimized design variables (E_r, β_r). It was also proposed to optimize each of the variables (E_r, β_r) independently into two successive steps: A) Young's modulus E_r, B) damping coefficient β_r.

Since the damping of rubber layer is introduced by damping coefficient β_r of proportional damping model that is coupled with E_{rD} by the relations (4)) and (5), there is necessary this coefficient to upgrade at each change of loss modulus of rubber E_{rD} by the relation $\beta_{ri} = E_{rDi}/\omega_1$.

As optimization method the gradient method was chosen when the values of the *i*th iteration step can be expressed as

$$E_{ri} = E_{ri-1} + (\partial E_r/\ \partial\omega_1)_{i-1}\Delta\omega_{1i-1}$$
$$E_{rDi} = E_{1Di-1} + (\partial E_{rD}/\ \partial b_1)_{i-1}\Delta b_{1i-1} \tag{5}$$

where $\Delta\omega_{1i-1}$, Δb_{1i-1} are given by difference of these constants in $(i-1)$the from the reference values. The gradients were substituted by differential expressions in the numerical scheme (5)

$$(\partial E_r/\ \partial\omega_1)_{i-1} = \frac{E_{r_{i-1}} - E_{r_{i-2}}}{\omega_{i-1} - \omega_{i-2}}$$
$$(\partial E_{rD}/\ \partial b_1)_{i-1} = \frac{E_{rD_{i-1}} - E_{rD_{i-2}}}{b_{i-1} - b_{i-2}} \tag{6}$$

Young's modulus E_r was gained by tuning of the first natural frequency to a required reference value in the step A). The undamped model of composed open sandwich beam with initial approximation of $E_r = 2$ MPa was used for this step. The values of eigenfrequency ω_1 and values of Young modulus E_r of the reference model and its initial and final approximations are presented in Table 1. Together with values of E_r the relative error $\Delta\ E_r$ of the approximations are mention in Table 1, too. The results of computational parameters E_r, $\partial E_r/\partial\omega_1$ and cost function $\Delta\omega_1$ during iteration process of optimization are graphically depicted in the Fig. 2.

The damping coefficient β_r was gained by tuning of a modal damping constant b_1 to its reference value in the next B step. Hereat the value of E_r was retained from the step A). The damped model of composed beam was used for this step. The values of damping constant b_1, eigenfrequency ω_1, damping ratio ζ, Young modulus E_r, damping coefficient β_r, loss modulus E_{rD} of reference model and its initial and end approximations are presented in Table 2. The values of relative error $\Delta\beta_r$ for the approximations are mention in the Table 2, too. The results of computational parameters damping coefficient β_r, loss modulus of rubber E_{rD}, gradient $\partial E_{rD}/\partial b_1$ and cost function Δb_1 during iteration process of optimization are graphically depicted in the Fig. 3.

The graphical dependences of Young modulus E_r (Fig. 2) and damping coefficenet β_r (Fig. 3) show that material constants converge very fast to their required reference values (red dashed lines). The very low values of the relative errors of their final estimations (Tables 1 and 2) proves the accuracy of the optimization.

The results of the optimization prove the correctness and accuracy of our proposal for rubber constant identification. It can be seen that by the proper model of rubber mass we can get quite "close" estimation of eigenfrequency of the composed beam. Then in the vicinity of eigenfrequency ω_1 its dependence on the

Please cite this article as: P. Šulc et al., Pre-stressed rubber material constant estimation for resilient wheel application, Advances in Engineering Software (2016), http://dx.doi.org/10.1016/j.advengsoft.2016.07.009