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Modeling large-deforming fluid-saturated porous media using an Eulerian incremental formulation

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ABSTRACT

The paper deals with modeling fluid saturated porous media subject to large deformation. An Eulerian incremental formulation is derived using the problem imposed in the spatial configuration in terms of the equilibrium equation and the mass conservation. Perturbation of the hyperelastic porous medium is described by the Biot model which involves poroelastic coefficients and the permeability governing the Darcy flow. Using the material derivative with respect to a convection velocity field we obtain the rate formulation which allows for linearization of the residuum function. For a given time discretization with backward finite difference approximation of the time derivatives, two incremental problems are obtained which constitute the predictor and corrector steps of the implicit time-integration scheme. Conforming mixed finite element approximation in space is used. Validation of the numerical model implemented in the SfePy code is reported for an isotropic medium with a hyperelastic solid phase. The proposed linearization scheme is motivated by the two-scale homogenization which will provide the local material poroelastic coefficients involved in the incremental formulation.

1. Introduction

Within the small strain theory, behavior of the fluid-saturated porous materials (FSPM) was described by Biot in [3] and further developed in [4]. It has been proved (see *e.g.* [1,2] cf. [41]) that the poroelastic coefficients obtained by Biot and Willis in [6] hold also for quasistatic situations, cf. [11]. In the dynamic case the inertia forces cannot be neglected, however, for small fluid pressure gradients leading to slow interstitial flows in pores under slow deformation and within the limits of the small strain theory, the effective poroelasticity coefficients of the material describing the macroscopic behavior can be obtained by homogenization of the static fluid-structure interaction at the level of micropores, see also [40].

Behavior of the FSPM at finite strains has been studied within the mixture theories [8,9] and also using the macroscopic phenomenological approach [5,17]. The former approach has been pursued in works which led to the theory of porous media (TPM) [19,23,25], including elastoplasticity [7], or extended for multiphysics problems, see *e.g.* [31]. Micromechanical approaches based on a volume averaging were considered in [22,24]. The issue of compressibility for the hyperelastic solid phase was considered in [26], cf. [21]. A very thorough revision of the viscous flows in hyperelastic media with inertia effects was given in [14]. There is a vast body of the literature devoted to the numerical modelling of porous media under large deformation, namely in the context of geomechanical applications, thus, taking into account plasticity of the solid phase [13,33,35]. Although the fluid retention in unsaturated media is so far not well understood in the context of deterministic models, a number of publications involve this phenomenon in computational models [32,44]. Concerning the problem formulation, three classical approaches can be used. Apart of the total Lagrangian (TL) formulations employed in the works related to the TPM [19,23,25], the updated Lagrangian (UL) formulation has been considered in [30,32,42]. As an alternative, the ALE formulation provides some advantages, like the prevention of the FE mesh distorsions, cf. [33], cf. [11]. Transient dynamic responses of the hyperelastic porous medium has been studied in [30], within the framework of the UL formulation with the linearization developed in [7]. In [42] the model has been enriched by the hydraulic hysteresis in unsaturated media.

It is worth to recall the generally accepted open problem of “missing one more equation” in the phenomenological theories of the FSPM. This deficiency can be circumvented by upscaling. For this, in particular, the homogenization of periodic media can be used, although several other treatments have been proposed, see

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e.g. [27] where the Biot's theory was recovered within the linearization concept. The conception of homogenization applied to upscale the Biot continuum in the framework of the updated Lagrangian formulation has been proposed [39], cf. [37]; recently in [11], where the ALE formulation was employed and some simplification were suggested to arrive at a computationally tractable two-scale problem. In [38] we described a nonlinear model of the Biot-type poroelasticity to treat situations when the deformation has a significant influence on the permeability tensor controlling the seepage flow and on the other poroelastic coefficients. Under the small deformation assumption and using the first order gradient theory of the continuum, the homogenized constitutive laws were modified to account for their dependence on the macroscopic variables involved in the upscaled problem. For this, the sensitivity analysis known from the shape optimization was adopted.

In this paper we propose a consistent Eulerian incremental formulation for the FSPM which is intended as the basis for multiscale computational analysis of FSPM undergoing large deformation. In the spatial configuration, we formulate the dynamic equilibrium equation using the concept of the effective stress involved in the Biot model. The fluid redistribution is governed by the Darcy flow model with approximate inertia effects, and by the mass conservation where the pore inflation is controlled through the Biot stress coupling coefficients. The compressibility of the solid and fluid phases are respected by the Biot modulus. The solid skeleton is represented by the neo-Hookean hyperelastic model employed in [30], although any finite strain energy function could be considered. First, the rate formulation, *i.e.* the Eulerian formulation, is derived by the differentiation of the residual function with respect to time using the conception of the material derivative consistent with the objective rate principle. The time discretization leads to the updated Lagrangian formulation whereby the out-of-balance terms are related to the residual function associated with the actual reference configuration. Although, in this paper, we do not describe behavior at the pore level, the proposed formulation is coherent with the homogenization procedure which can provide the desired effective material parameters at the deformed configuration. In relationships with existing works [37,39] and [11], this is an important aspect which justifies the proposed formulation as an improved modelling framework for a more accurate multi-scale computational analysis of the porous media.

The paper is organized, as follows. In Section 2 we present a model of the hyperelastic FSPM described in the spatial configuration and formulate the nonlinear problem. The inertia effects related to both phases are accounted for in Definition 1, although in Definition 2 we consider the simplified dynamics. Section 3 constitutes the main part of the paper. There we introduce the rate Eulerian formulation using the above mentioned differentiation of the residual function. Then we explain the time discretization which leads to the finite time step incremental formulation suitable for the numerical computation using the finite element (FE) method. In this procedure, we neglect the inertia terms associated with the fluid seepage in the skeleton. Definitions 4 and 5 provide two approximate incremental formulations related the predictor and corrector steps, respectively. In Section 4, we provide a numerical illustration and validation of the incremental formulation using 2D examples considered in the work [30], namely the 1D compression test and the consolidation of a partially loaded layer. For the validation we use the comparison between responses obtained by the proposed model with those obtained by linear models. Besides inspection of the displacement fields, we show the influence of the time step length on the fluid content, strain energy and the dissipation.

Basic notations. Through the paper we shall adhere to the following notation. The spatial position x in the medium is specified through the coordinates (x_1, x_2, x_3) with respect to a Cartesian reference frame. The Einstein summation convention is used which stipulates implicitly that repeated indices are summed over. For any two vectors $\boldsymbol{a}$, $\boldsymbol{b}$, the inner product is $\boldsymbol{a} \cdot \boldsymbol{b}$, for any two 2nd order tensors $\boldsymbol{A}$, $\boldsymbol{B}$ the trace of $\boldsymbol{A}\boldsymbol{B}^T$ is $\boldsymbol{A} : \boldsymbol{B} = A_{ij}B_{ij}$. The gradient with respect to coordinate x is denoted by ∇. We shall use gradients of the vector fields; the symmetric gradient of a vector function $\boldsymbol{u}$, *i.e.* the small strain tensor is $\boldsymbol{e}(\boldsymbol{u}) = 1/2[(\nabla\boldsymbol{u})^T + \nabla\boldsymbol{u}]$ where the transpose operator is denoted by the superscript ^{T}. The components of a vector $\boldsymbol{u}$ will be denoted by u_i for $i = 1, \dots, 3$, thus $\boldsymbol{u} = (u_i)$. To understand the tensorial notation, in this context, $(\nabla\boldsymbol{v})_{ij} = \partial_j v_i$, so that $((\nabla\boldsymbol{v})^T)_{ij} = \partial_i v_j$, where $\partial_i = \partial/\partial x_i$. By $\overline{D}$ we denote the closure of a bounded domain D. $\boldsymbol{n}$ is a unit normal vector defined on a boundary ∂D, oriented outwards of D. The real number set is denoted by $\mathbb{R}$. Other notations are introduced through the text.

2. Model of large deforming porous medium

Behavior of the FSPM is governed by the equilibrium equations and the mass conservation governing the Darcy flow in the large deforming porous material. Initial configuration associated with material coordinates X_i, $i = 1, 2, 3$ and with domain Ω_0 is mapped to the spatial (current) configuration at time t associated with spatial coordinates x_i and with domain $\Omega(t)$. Thus, $x_i = \varphi_i(t, X)$ where $\boldsymbol{\varphi} = (\varphi_i)$ is continuously differentiable. By $\boldsymbol{F} = (F_{ij})$ we denote the deformation gradient given by $F_{ij} = \partial x_i/\partial X_j = \delta_{ij} + \partial u_i/\partial X_j$.

2.1. Governing equations – Biot model

We shall first introduce the governing equations for the problem of fluid diffusion through hyperelastic porous skeleton. These involve the Cauchy stress tensor $\boldsymbol{\sigma} = (\sigma_{ij})$, the displacement field $\boldsymbol{u} = (u_i)$, the bulk pressure[1] p and the perfusion velocity $\boldsymbol{w} = (w_i)$ which describes motion of the fluid relative to the solid phase. The constitutive laws for $\boldsymbol{\sigma} = (\sigma_{ij})$ reads as

$$\boldsymbol{\sigma} = -p\boldsymbol{B} + \boldsymbol{\sigma}^{\text{eff}}(\boldsymbol{u}), \tag{1}$$

$$\boldsymbol{\sigma}^{\text{eff}} = J^{-1}[\mu\boldsymbol{b} + (\lambda \ln J - \mu)\boldsymbol{I}], \tag{2}$$

where p is the pore fluid pressure, $J = \det\boldsymbol{F}$ expresses the relative volume change of the skeleton, $\boldsymbol{b} = \boldsymbol{F}\boldsymbol{F}^T$ is the left deformation tensor, $\boldsymbol{B} = (B_{ij})$ is the Biot coupling coefficient. Further, by "eff" we refer to the effective (hyperelastic) stress, which is related to a strain energy function. Here we consider a neo-Hookean-type material model employed in [30], which is parameterized by μ and the Lamé constant λ; for a small strain approximation, $\mu' := \mu - \lambda \ln J$ expresses the shear modulus.

The inertia of the solid and fluid phases is expressed by the skeleton acceleration $\boldsymbol{a}^s$ and by the relative acceleration of the fluid

$$\boldsymbol{a}^R = \dot{\widehat{\left(\frac{\boldsymbol{w}}{\phi}\right)}}, \quad \boldsymbol{w} = \phi(\boldsymbol{v}^f - \boldsymbol{v}^s), \tag{3}$$

where $\boldsymbol{v}^f$, $\boldsymbol{v}^s$ are the solid and fluid phase velocities, respectively.[2] It is now possible to establish the Darcy law extended for dynamic flows in the moving porous skeleton. According to Coussy [16]

$$\begin{aligned}\boldsymbol{w} &= -\boldsymbol{K}(\nabla p - \rho_f(\boldsymbol{f} - \boldsymbol{a}^f - \boldsymbol{a}))\\ &\approx -\boldsymbol{K}(\nabla p - \rho_f(\boldsymbol{f} - \boldsymbol{a}^s - \alpha\boldsymbol{a}^R))\end{aligned} \tag{4}$$

[1] We adhere to the standard sign convention: the positive pressure means the negative volumetric stress induced by compression.

[2] This expression for $\boldsymbol{a}^R$ is merely approximate since the dot means the material derivative with respect to the skeleton, for details see *e.g.* [14].