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Research paper

Sound transmission analysis of plate structures using the finite element method and elementary radiator approach with radiator error index

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A B S T R A C T

In this paper, an accurate and efficient numerical method for sound transmission analysis is presented. As an alternative to conventional numerical methods, such as the Finite Element Method (FEM), Boundary Element Method (BEM) and Statistical Energy Analysis (SEA), the FE-ERA method, which combines the FEM and Elementary Radiator Approach (ERA) is proposed. The FE-ERA method analyzes the vibrational response of the plate structure excited by incident sound using FEM and then computes the transmitted acoustic pressure from the vibrating plate using ERA. In order to improve the accuracy and efficiency of the FE-ERA method, a novel criterion for the optimal number of elementary radiators is proposed. The criterion is based on the radiator error index that is derived to estimate the accuracy of the computation with used number of radiators. Using the proposed criterion a radiator selection method is presented for determining the optimum number of radiators. The presented radiator selection method and the FE-ERA method are combined to improve the computational accuracy and efficiency. Several numerical examples that have been rarely addressed in previous studies, are presented with the proposed method. The accuracy and efficiency of the proposed method are validated by comparison with the results of the three dimensional (3D) FEM structure-acoustic interaction models.

1. Introduction

The sound insulation performance of a plate structure is evaluated by sound transmission loss (STL), which is the power ratio between incident and transmitted sound. Generally, the STL is proportional to the mass of the plate structure [1,2]. Because the amount of mass usage in a plate for sound insulation is highly related to performances of mechanical systems such as fuel efficiency, pollution and transportation costs, an accurate prediction of the plate STL is essential.

Previously, analytical and numerical methods have been developed to analyze the STL of the plate structure. The analytical methods are classified according to the infinite and finite plate models [3–9]. For existing method based on the infinite plate model, there is no established accurate method to explain STL at the low frequency range because this model ignores the flexible modes of the plate [3–5]. On the other hand, the finite plate model considers the influence of the flexible modes by using the mode superposition method [6–9]. However, this method is limited to simple plate structures that can express the mode shape in the analytical form. There are complicated plate structures in practical applications, but analytical expressions of the mode shapes are very complicated. In this respect, the analytical sound transmission analysis method has limitations in real-life mechanical systems.

On the other hand, numerical methods based on FEM, BEM and SEA could handle complicated plate structures that are limited in the analytical methods. However, these conventional numerical methods require significant computation costs and modeling efforts because the methods require a 3D structural-acoustic interaction model and couplings of the different numerical schemes, such as FEM-BEM and FEM-SEA [10–14].

As an alternative of the previous sound transmission analysis, an efficient and accurate method is developed in this paper. Following the traditional analytical model, the sound transmission phenomenon could be formulated as sound radiation from a vibrating plate that is excited by incident sound [7–9]. Analysis of the vibrational response of the plate is straightforward; it follows the FEM procedure with no restriction on the plate geometry. Most of the computation cost and efforts come from the interaction between the plate and ambient air. To alleviate the difficulties in

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the interaction procedure, we employed an elementary radiator approach that is known to be an efficient method for computing sound radiation [15–20]. The ERA considers the sound radiation from a plate as the sum of the sound radiation from the large number of radiators mounted on the plate. In this way, the radiated acoustic pressure is expressed by a simple algebraic formulation that is computationally efficient. Taking advantages both of the ERA and FEM, a sound transmission analysis method (i.e., FE-ERA method) is developed. Compared with the previous sound transmission analysis, the FE-ERA method is computationally efficient because it does not requires the 3D structural-acoustic interaction model or coupling of the different numerical schemes.

In second part of this paper, a further improvement of the developed FE-ERA method is addressed. The computational efficiency and accuracy of the FE-ERA method significantly depend on the number of radiators. Although a lot of studies have been conducted on the ERA in vibro-acoustic applications, the required number of radiators for efficiency and accuracy is still unclear [15–20]. In the reference [20], Chandra et al. have stated that the characteristics length of the radiators should be much smaller than the minimum acoustic wave length for accurate computational results. Following this rule, the required number of radiators vary according to the frequencies of interest. Thus, this rule is inefficient when the computation is conducted on the wide range of frequencies. In this regard, we present a novel criterion for the required number of radiators. The criterion is based on the radiator error index that is derived in order to estimate the accuracy of the considered number of radiators. Using the presented criterion, a radiator selection method is proposed for adaptively determining the required number of radiators. Based on the proposed radiator selection method, the efficiency of the FE-ERA method can be further improved.

This paper consists of 6 sections, including the introduction. The background of the sound transmission through a finite plate structure is reviewed in Section 2. The modeling of sound transmission problem is explained in Section 3. In Section 4, the radiator error index is derived and the radiator selection method is explained. Several numerical examples are solved using the FE-ERA method in Section 5. The accuracy and efficiency of the proposed method are validated by comparison with the result of the 3D FEM model. Finally, Section 6 concludes the paper.

2. Background: sound transmission through a finite plate

In this section, the background of sound transmission through a finite plate is reviewed. If sound is incident on the finite plate, the plate vibrates by the acoustic pressure and the vibrating plate reflects and transmits sound by exciting ambient air. Fig. 1 shows the sound reflection and transmission phenomena of the plates.

The sound insulation performance of the plate is evaluated by the sound transmission loss (*STL*), which is a ratio between the incident sound power (Π_i) and transmitted sound power (Π_t) [1,2]. The STL is defined as

$$STL = 10\log_{10}\frac{\Pi_i}{\Pi_t},$$

where,

$$\Pi_t : \textit{Transmitted Sound Power},$$
$$\Pi_i : \textit{Incident Sound Power}. \tag{1}$$

The STL of a finite plate is classified by three regions, as shown in Fig. 2. The stiffness controlled region is the low frequency region below the 1st natural frequency of the plate. At the stiffness controlled region, the STL rapidly decreases as the frequency increases. The resonance controlled region is the intermediate region between the stiffness and mass controlled regions. At the resonance controlled region, the STL is decreased or increased by the resonances and anti-resonances of the plate. The mass controlled region is the frequency region above the resonance controlled region. At the mass controlled region, the STL is proportional to the mass and frequency of incident sound. The STL of the mass controlled region is governed by the mass law [1,2]. For normally incident sound the mass law is stated as

$$STL_{Mass\ law} = 20\log_{10}\left(1 + \frac{\pi f m}{\rho_0 c_0}\right),$$

where,

$$f : \textit{Frequency of incident sound}\ [Hz],$$
$$m : \textit{Area mass density of plate}\ [kg/m^2],$$
$$\rho_0 : \textit{Mass density of air}\ \left[kg/m^3\right],$$
$$c_0 : \textit{Speed of sound}\ [m/s]. \tag{2}$$

Although the mass law is very simple and compact, it is only valid for large simple plates because it is derived from the infinite and rigid plate assumption [1,2,5]. Therefore, the mass law could not accurately predict the STL of general finite plate structures.

3. Modeling of sound transmission through a finite plate

In this section, the modeling of sound transmission through a finite plate structure is explained in detail. The sound transmission of the finite plate is formulated using FEM and ERA. To the best of our knowledge, the method combining FEM and ERA is applied to the sound transmission problem for the first time in this paper.

3.1. Vibro-acoustic formulation

To model sound transmission through a finite plate, the acoustic wave with angle θ is considered to be incident on the baffled plate, as shown in Fig. 3. Here, $\mathbf{r}$ and $\mathbf{r_0}$ are the position vectors in 3D and 2D space on the plate, respectively.

The equation of the motion of the plate excited by incident sound can be expressed as follows [1,2,6–9],

$$\left(D\nabla^4 - \rho_s\omega^2\right)w(x,y)$$
$$= 2\exp\left[-j(k_x x + k_y y)\right] + \frac{\rho_0\omega^2}{\pi}\int_{S_0}\frac{\exp(-jkR)}{R}w(x_0,y_0)dS_0,$$
$$R(x,y) = \left[(x-x_0)^2 + (y-y_0)^2\right]^{\frac{1}{2}} \tag{3}$$

For the sake of simplicity the derivation of Eq. (3) is omitted in this paper. For the detail of derivation procedure, please refer to Ref. [7]. In the Eq. (3), D, ρ_s, and ∇^4 are the bending rigidity of the plate, area mass density and Biharmonic operator, respectively. Variable w is the bending displacement of the plate and ω, k, k_x, and k_y are the angular frequency, wave number, and x and y components of the wave vector for incident sound, respectively. The notation ρ_0 is the mass density of the ambient air, and S_0 is the area of the plate. The notation R is the distance between two points (i.e., (x, y) and (x_0, y_0)) in the space. In Eq. (3), the left hand side terms represent the vibration of the plate and the right hand side terms are the excitation force of the incident sound and radiation resistance of ambient air, respectively. Coefficient 2 in front of the excitation force term results from blocked acoustic pressure.

For a flat plate structure, the transmitted acoustic pressure (p_t) can be expressed using Rayleigh's integral as

$$p_t(x,y) = -\frac{\rho_0\omega^2}{2\pi}\int_{S_0}\frac{\exp(-jkR)}{R}w(x_0,y_0)dS_0. \tag{4}$$

The incident sound power (Π_i), transmitted sound power (Π_t) and transmission coefficient (τ) are defined as follows

$$\Pi_i(\theta,\omega) = \frac{1}{2}\mathrm{Re}\left\{\int_S p_i v^* dS\right\} = \frac{\cos(\theta)S}{2\rho_0 c_0}|p_i|^2, \tag{5}$$